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Information Based Analysis of Fission Source Correlation

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Abstract

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Information Based Analysis of Fission Source Correlation

Brian Kiedrowski (Univ. Wisconsin) & Forrest Brown (LANL)

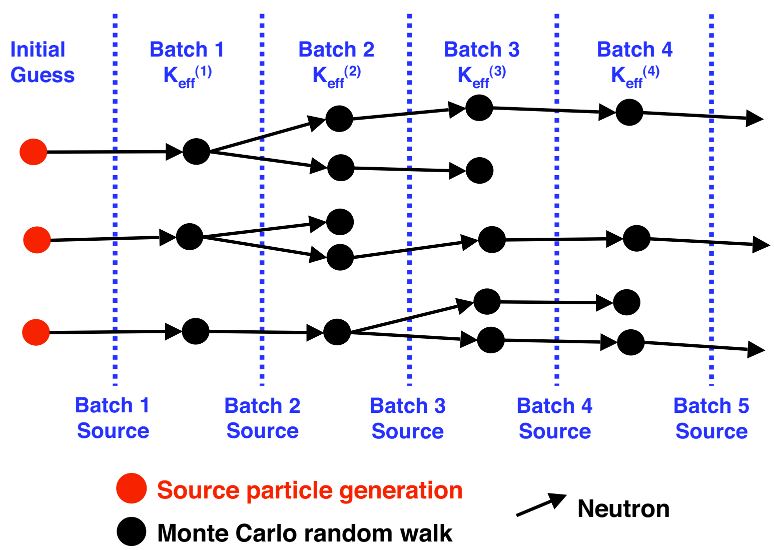
The variances estimated by Monte Carlo codes in k-eigenvalue calculations are underpredicted due to inter-cycle correlation between fission sources. The mutual information serves as a diagnostic to measure the correlation between fission source distributions in different cycles. There is a definite observed relationship between the variance bias and the mutual information of the source distributions. Furthermore, using the mutual information in conjunction with the Wielandt method shows how effective a particular Wielandt shift is at removing variance bias. Finally, the dominance ratio and the mutual information are related to MacMillan's correction.

Overview

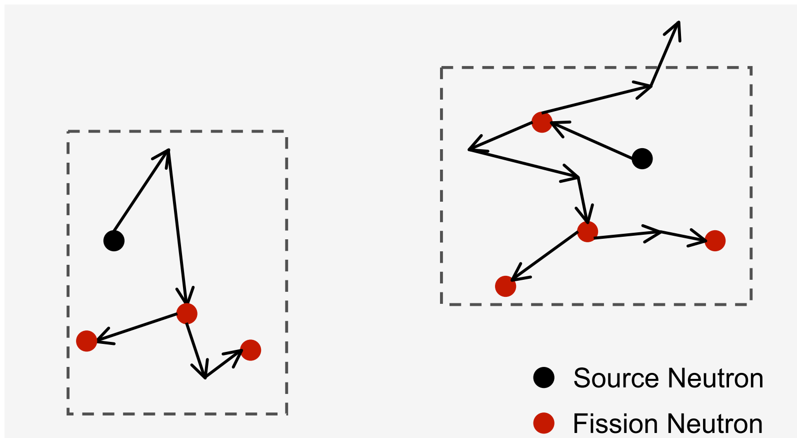
- Fission source correlation
- Mutual Information
- Application to Wielandt Method
- Relationship to Dominance Ratio

MCNP Criticality Calculations

- Power iteration method:



Fission Source Correlation



Fission neutrons in following generations tend to be near the source neutron in the previous generations.

Fission Source Correlation

- Correlation always positive
- Causes underprediction of variance:

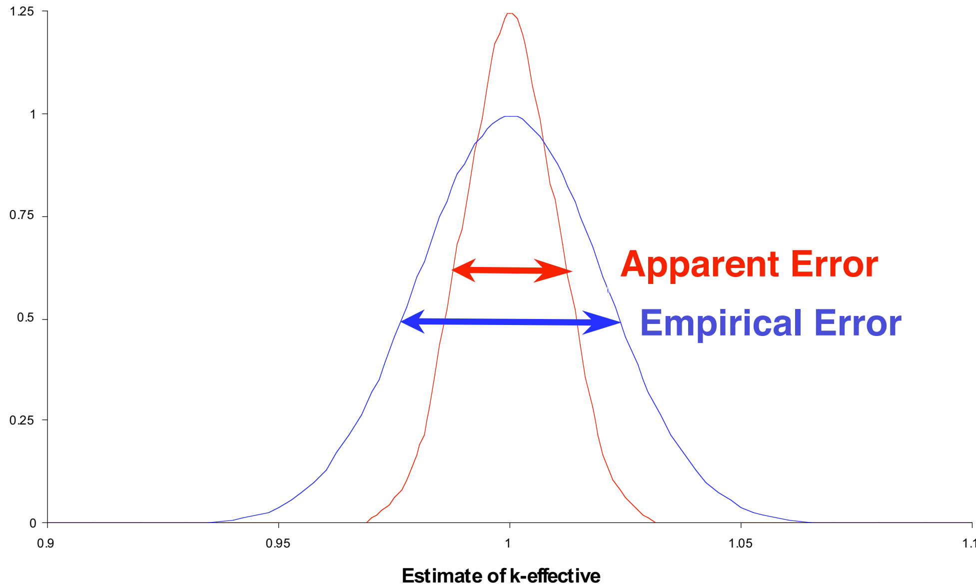
Apparent Variance:

$$\tilde{\sigma}_{\bar{x}}^2 = \frac{1}{N}\left[\frac{1}{N-1}\sum_{i=1}^{N} x_i^2 - \bar{x}^2\right]$$

Empirical Variance:

$$\sigma_{\bar{x}}^2 = \tilde{\sigma}_{\bar{x}}^2 + \sum_{i,j} r_{ij}$$

Variance Bias



Variance Bias

- **Most Monte Carlo codes give apparent error**

- **Should give empirical error**
  - MacMillan's factor (highly conservative)
  - Lag coefficients (subject to stochastic noise)
  - Brute force method (very time consuming)
    - **Average numerous (25-100+) results from calculations with different random number seeds.**

Variance Bias

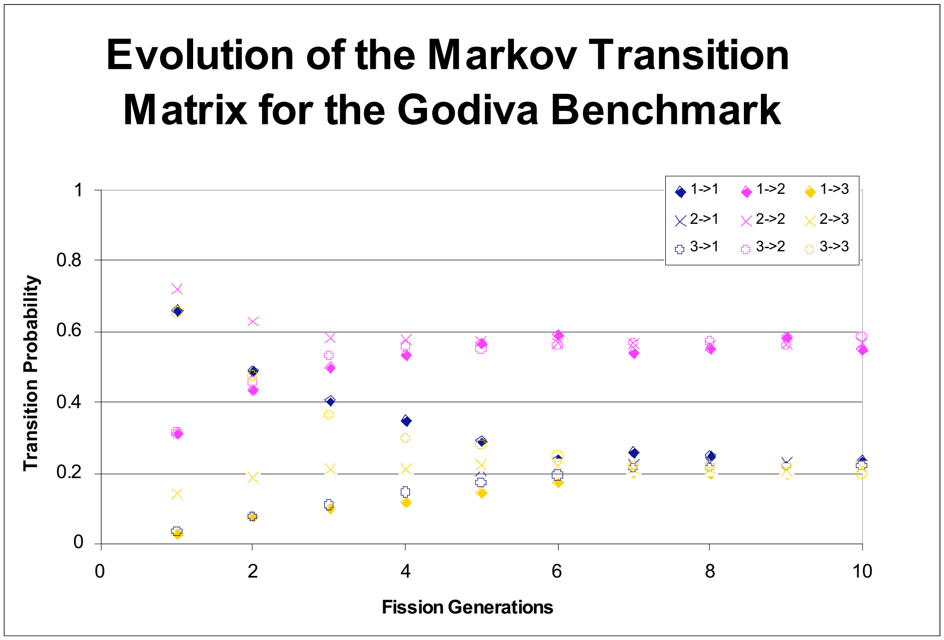
- **Small impact on k-effective (5-20%).**
- **Significant impact on local tallies:**

Benchmark	Tally $\sigma / \tilde{\sigma}$
Godiva	1.43
Two Cylinder	4.56
B&W Core	3.34
Checkerboard	3.97

Ergodicity of Fission Source

- **Intercycle correlation should decrease with separation distance.**
- **True if fission source transition is an <u>ergodic</u> Markov chain.**
- **Ergodic: Loss of memory of initial state after many transitions.**
  - Fission matrix for *j* transitions step measures.
  - Final state independent of initial state.

Ergodicity of Fission Source



Ergodicity of Fission Source

- **Information about initial state lost each transition.**
- **Loss of information implies loss of correlation**
  – Final state independent of initial state
- **Since correlation leads to variance bias, can some simple measure help estimate this?**

Mutual Information

- Measures information gained about an unmeasured result from the measurement of another.

$$I(X,Y) = H(Y) - H(Y \mid X)$$

- Measured from fission matrix elements:

$$I = \sum_i \sum_j F_{ij} \log\left(\frac{F_{ij}}{f_i f_j}\right)$$

Mutual Information

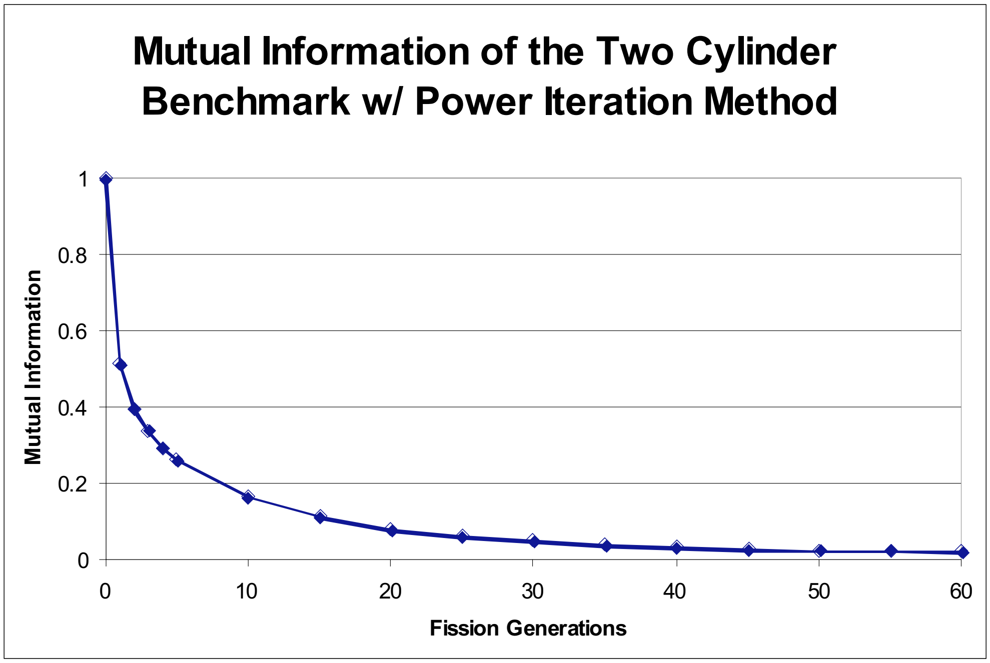
- Measures correlation between fission populations in one generation to the next.

- As separation distance increases to infinity, mutual information goes to zero.

$$F_{ij} = f_i f_j$$

$$I = \sum_i \sum_j f_i f_j \log\left(\frac{f_i f_j}{f_i f_j}\right) = 0$$

Mutual Information Decay



Mutual Information Convergence

- Mutual information sensitivity to mesh size for Godiva problem.

Mesh Spacing	Mutual Information
3x3x3	0.233
4x4x4	0.225
5x5x5	0.217
6x6x6	0.211

Mutual Information Convergence

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- Mutual information sensitivity to batch size for Godiva problem (3x3x3 mesh).

Batch Size (histories)	Mutual Information
10^2	0.558
10^3	0.327
10^4	0.241
10^5	0.230
10^6	0.229

Wielandt Method

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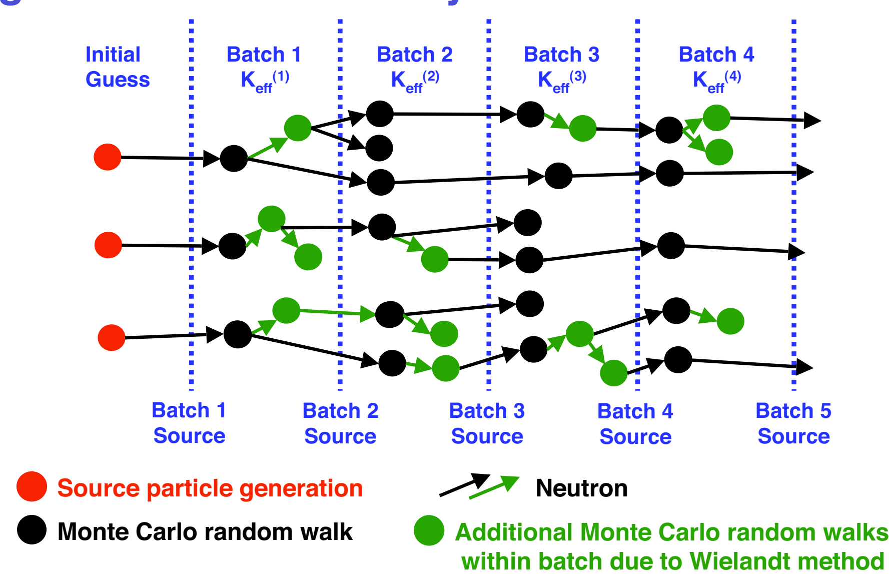
- Stochastically extends the number of generations within a cycle.



Wielandt Method

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- **Longer fission chains**
  - Decrease the number of cycles required for convergence (same CPU time).
  - Reduces bias in error estimates.

- **Average chain length:**

$$L = 1 + \frac{k_0}{\Delta_W}$$

Wielandt Method Results

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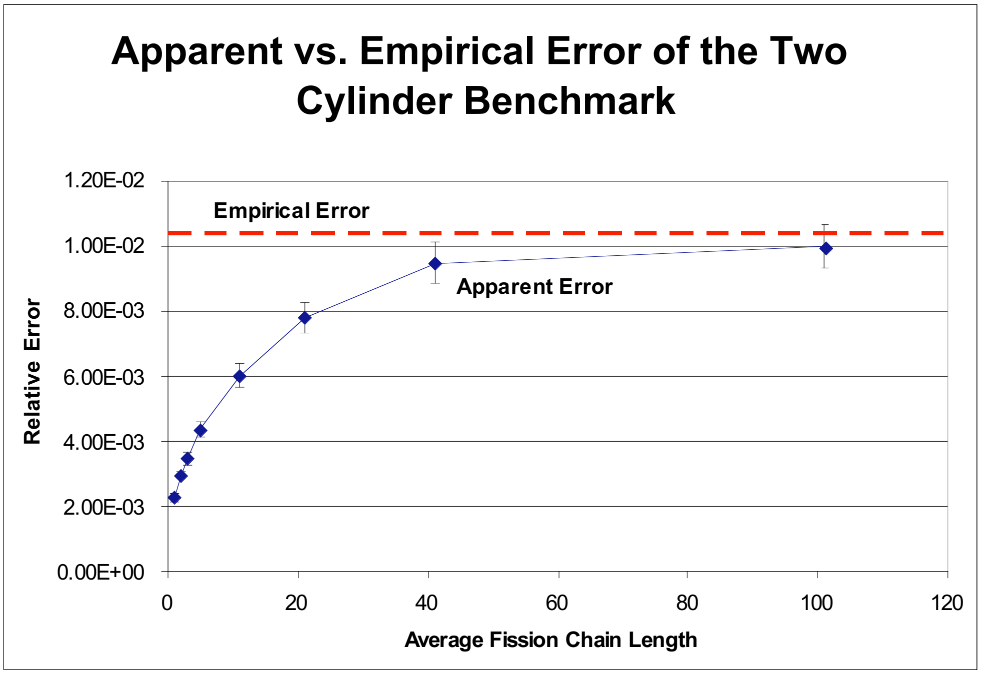

Wielandt Method Results

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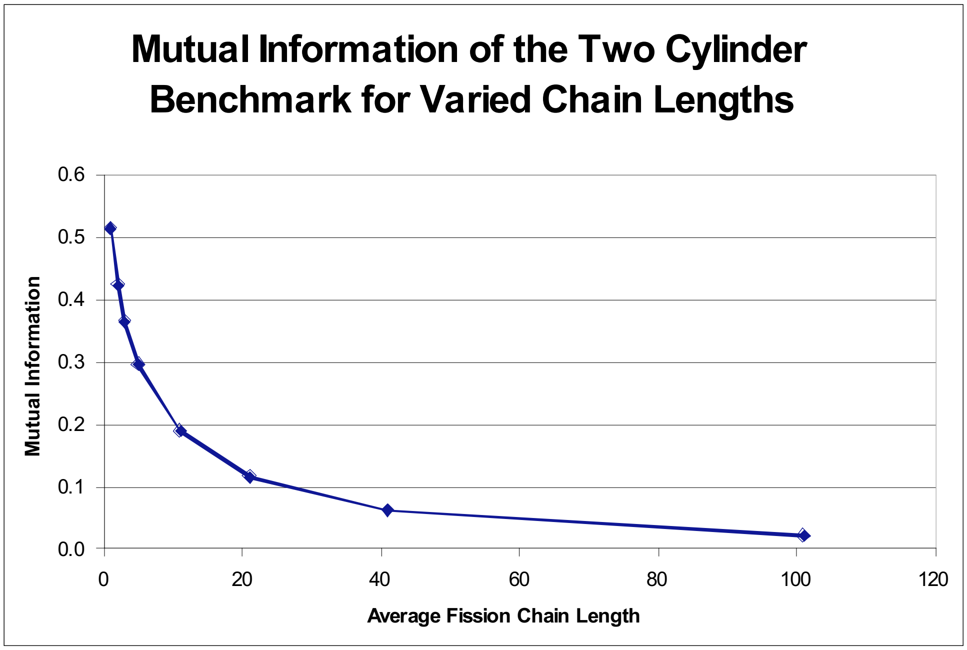

Wielandt Method Results

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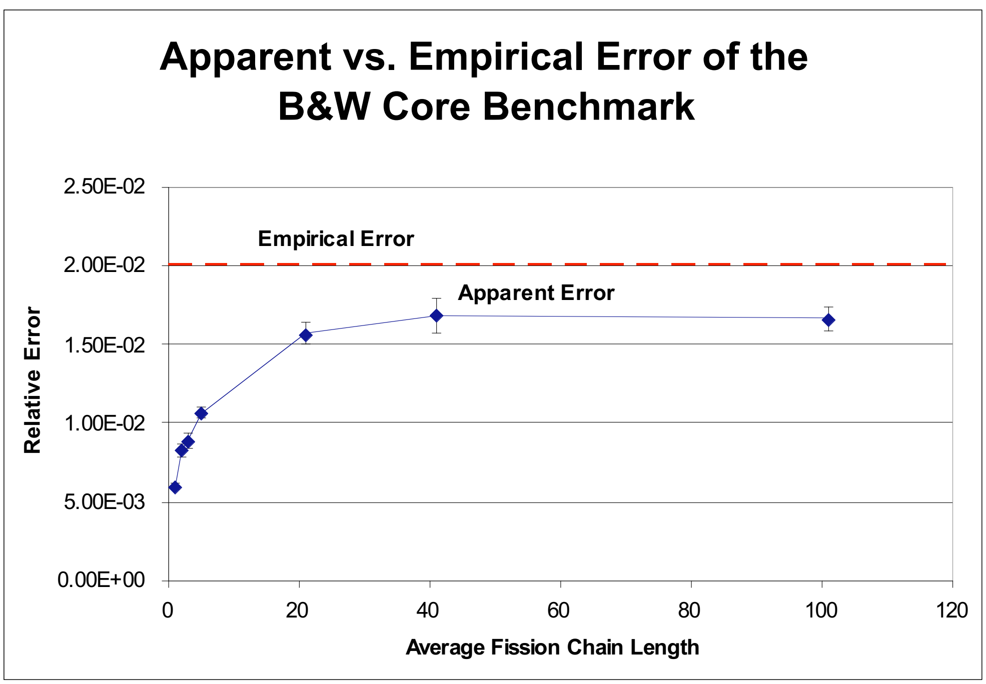
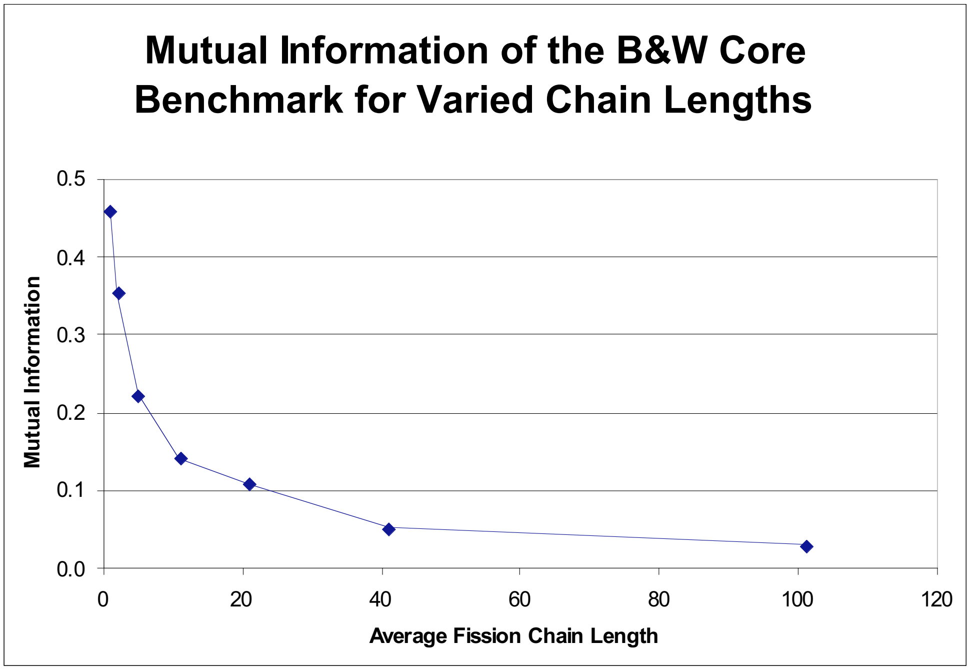
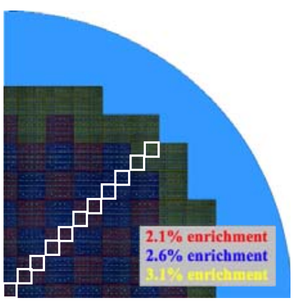


Wielandt Method Results



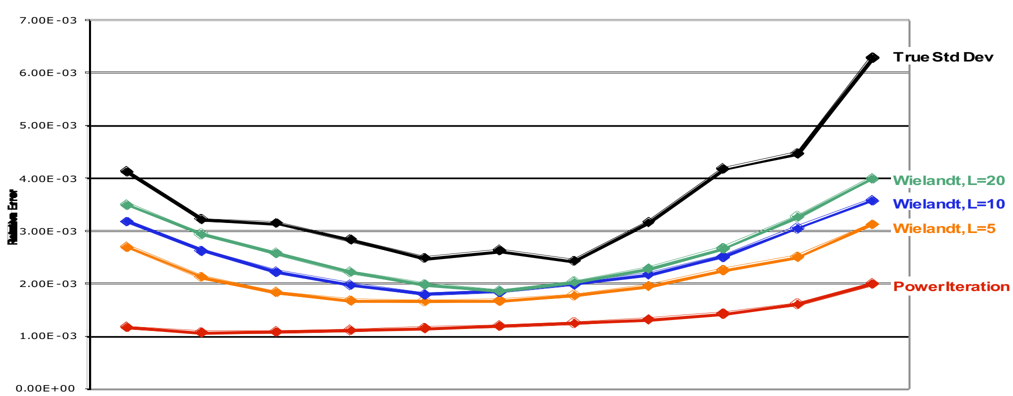
Wielandt Method Results



2D quarter-core PWR (Nakagawa & Mori model)

+ Explicit fuel pins & rod channels, 17x17
+ 120 M active neutrons for each calculation
+ Tally fission rates in each quarter-assembly

Plot relative error in quarter-assemblies along diagonal



Observations

- **Mutual information seems to provide a good measure of variance bias in MOST problems.**
- **B&W Core benchmark shows disagreement.**
  - More investigation needed.
- **Variance bias is local in nature.**
  - Mutual information is global.
  - No clear relationship to k-effective.

Dominance Ratio

- **Measures effect of higher order modes.**
- **Stochastic fluctuations excite higher modes.**
- **Higher dominance ratio means slower decay, and more correlation.**

$$\rho = \frac{k_1}{k_0}$$

Dominance Ratio vs. Mutual Information

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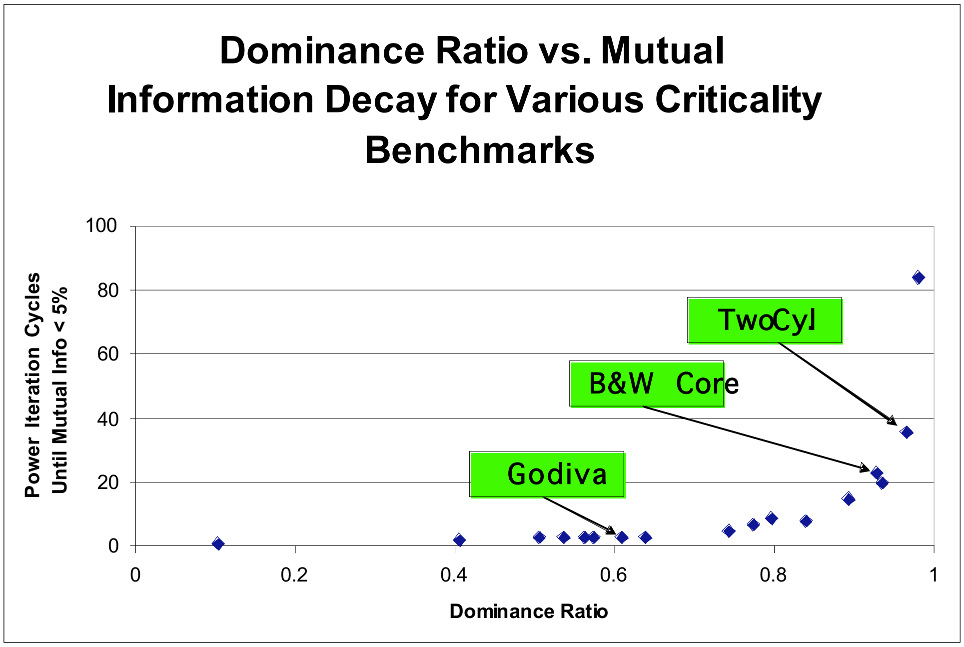

Relation to MacMillan Factor

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- **MacMillan's factor:**

$$M = 1 + \frac{2r}{1 - \rho}$$

- **Mutual information curve appears to follow similar asymptotic trend.**
  - Suggestive, but more analysis needed.

Conclusions & Future Work

- **Mutual information is a useful diagnostic to get general feel of correlation effects.**
  – No definitive connection to variance bias.
- **Very suggestive for many problems.**
- **Still issues:**
  – Connection to global k-effective estimates
  – B&W core convergence issues
  – Local variations

Questions?