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Title: ANALYTIC VERIFICATION OF NON-BOLTZMANN PROBLEMS

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Analytic Verification of Non-Boltzmann Problems

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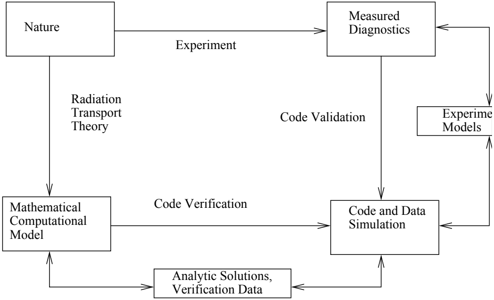
Motivation
Introduction
Analytic Verification
Probablistic approach
Pulse-height Spectra
Verification Test Problem
Verification Problem Solution
Pulse Height Results
Photons
Photons and Electrons
Conclusions

Motivation for this work

- Upcoming release of MCNP 5.1.50
    - Major feature: allows pulse-height tallies with all of MCNP's variance reduction
    - Pulse-height tallies are non-Boltzmann
    - Traditional methods of verification do not apply
    - Very easy to make a mistake when developing variance reduction theory for pulse-height tallies
        - How do you **know** that you did not make a mistake?

1. Motivation
2. Introduction
    - Analytic Verification
    - The Probablistic Approach
3. Pulse-Height Spectra
4. Verification Test Problem
5. Verification Problem Solution
6. Pulse Height Results: Analytic and Calculated
    - Photons
    - Photons and Electrons
7. Conclusions

A Personal View of Computer Simulation of Radiation Transport



- Types of analytic code verification
  - Line-by-line examination of algorithm, Comparison to other codes, Regression test suite, Alternate problem representation
  - *High precision analytic results*

Generating high-precision analytic results

- Radiation transport codes begin with Boltzmann transport equation (BTE)
  - Transport is a function of 3-D space, energy, time, ...
- Deterministic techniques (eg. S_n)
  - discretize each variable, and 'moves' particles from one differential phase space to the next, estimates average behaviour, provides precise answers everywhere
- Monte Carlo:
  - 'moves' particles from event to event, no approximations in space, energy, time; provides exact physics, and calculates average answers locally
- Common approach for analytic verification for *both* methods simplify/eliminate terms in BTE
  - 1-D, single energy, isotropic scattering, steady-state
- Generate high-precision answers and compare calculations

Probablistic approach to analytic verification:

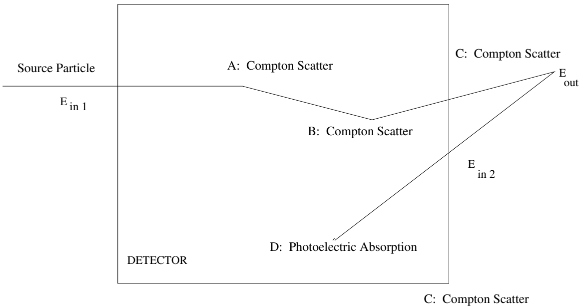
- New approach:
  - The main difficulty in verification of transport codes is in the complexity of nuclear data.
  - Solution: Simplify physical data (nuclear cross sections), leaving mathematical equations and code unaltered such that:
    - A limited set of occurrences exists (few?)
    - Probability of each occurrence can be calculated (easily?)
  - *Control the code by controlling the nuclear data!*
- Approach allows analytic verification of codes that is independent of solution technique and theoretical description:
  - deterministic vs. Monte Carlo, Boltzmann vs non-Boltzmann
- This approach is not widely used (or known?)

Pulse-Height Tallies

What are Pulse-Height tallies?

- Pulse-height tallies record energy deposited by source particle and all progeny
- Simulates ’perfect’ radiation detector

EXAMPLE HISTORY



Flux: each track scores a different energy

Pulse Height: tally total energy deposited from complete history

Energy Deposited = $(E_{in\ 1} + E_{in\ 2}) - E_{out}$

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  Analytic Verification
  Probablistic approach

Pulse-height Spectra

Verification Test Problem

Verification Problem Solution

Pulse Height Results
  Photons
  Photons and Electrons

Conclusions

Calculation of Pulse-Heights

- Monte Carlo is the *only* way to calculate this from first principles
  - Cannot be described by Boltzmann transport equation
  - Unlike other quantities (flux, current) which are calculated as particle exits or collides in a cell (Boltzmann-like)
    - Requires completion of all tracks for a history before the tally is made (joint PDFs)
    - Tally is done on a *collection* of particles
  - Boltzmann quantities (eg flux, current, etc) ignore any correlation between particles
- Requires microscopically correct data - energy must be conserved.

Verification Test Objectives

- Why are we interested in pulse-height tallies?
    - Pulse-height tallies have been available in MCNP since mid-80's
    - No analytic verification has been done
- Traditional analytic verification methods rely on BTE simplifications
    - Traditional methods cannot help
    - Probablistic approach can help
- Analytic verification problem must be:
    - Analytic verification problem is as physically realistic as possible (i.e non-trivial)
    - Analytically verify PHT (standard MCNP)
    - Analytically verify PHT with Variance Reduction - a major feature of next release!!
    - Ideally: also verify Boltzmann-type tallies in MCNP

The analytic test problem...

Definition of Test Problem

- Very clever isotopes defined by Ted Shuttleworth!!![1]
- Only gives examples of test cases.
  Scratched the surface of the complete solution.[2]
- Defined fictitious isotopes defining physics of "Middle Earth"[3]
  - If you can control the data, you can control the code...
  - Two cylinders: length of ln(2), radius ln(2)
  - Monodirectional point source along cylindrical axis: 3.2 MeV
  - Homogenous materials comprised of fictitious isotopes

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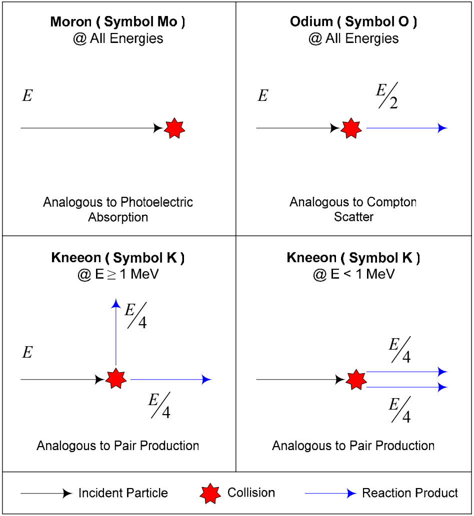
[1] Shuttleworth, T., “The Verification of Monte Carlo Codes in Middle Earth,” Proc. of 8th Int. Conf. Rad. Shield., Arlington, Tx (1994).

[2] Personal communication, (2002).

[3] J.R.R. Tolkien, The Lord of the Rings, Unwin Books London (1974).

Collision Physics



Moron (Symbol Mo)
@ All Energies

Analogous to Photoelectric Absorption

Odium (Symbol O)
@ All Energies

Analogous to Compton Scatter

Kneeon (Symbol K)
@ E ≥ 1 MeV

Analogous to Pair Production

Kneeon (Symbol K)
@ E < 1 MeV

Analogous to Pair Production

→ Incident Particle
✶ Collision
→ Reaction Product

Shuttleworth's Fictitious Isotopes

- Moron (chemical symbol: Mo)
  - Pure absorber
  - Analogous to **photoelectric effect**
  - 0.2 atom fraction
- Odium (chemical symbol: O)
  - Undeflected secondary particle with 1/2 initial energy
  - Analogous to **Compton scattering**
  - 0.3 atom fraction
- Kneeon (chemical symbol: Kn)
  - Produces 2 secondary particles with 1/4 initial energy
    - $<$ 1 MeV: both particles undeflected
    - $>$ 1 MeV: one particle undeflected, one particle 90^o scatter
  - Analogous to **pair production**
  - 0.5 atom fraction

Comments on problem details

- Σ_t = 1.0 cm^{-1}, Energy cutoff of 0.15 MeV (absorption)
- Sampling of Kneeon data must be correlated
- Collisions continuous in space
- Particles can multiply and produce progeny
- Forward and radial scattering; no backwards scattering
- Discrete energy deposition
- *Each possible collision can be analytically calculated using sophomore level nuclear engineering/physics**

* B.J. Adams, Reed College Phys. Dept., summer 2002

* M.S. Reed, TAMU Nucl. Eng. Dept, summer 2003,2004

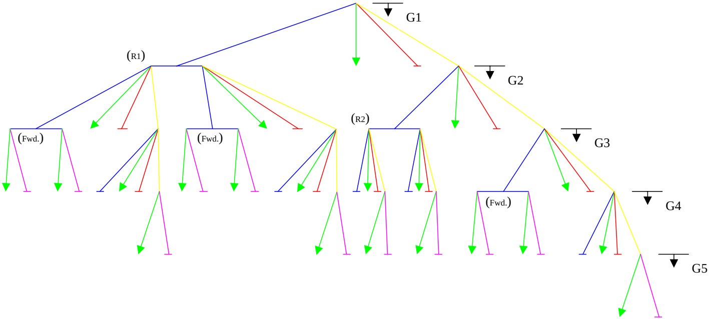
Scenario Tree

Interaction Key

🟥 Moron Collision	🟨 Odium Collision
🟦 Kneeon Collision	🟪 Arbitrary Collision
🟩 Exit	

Nomenclature

G#: Subsection of history tree corresponding to group #
(R1): Subsection of history tree representing first radial scatter track
(R2): Subsection of history tree representing second radial scatter track
(Fwd.): Location of a dual forward scatter in history tree

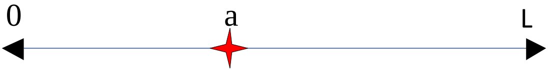


- 161 possible histories; 0 to 7 collisions possible
- 5 discrete energies

Calculating the Analytic Solution

- All probabilites are calculated using sophomore level physics:
    - $e^{-\Sigma x}$ is the probability of travelling a distance x without a collision, and Σdx is the probability of having a collision in dx
    - The probability of at least one collision between points a and b is:
$$\int_b^a e^{-\Sigma x} \Sigma dx = e^{-\Sigma(b-a)}$$
- Don't let this fool you...the devil is in the details

N Collisions followed by an escape:



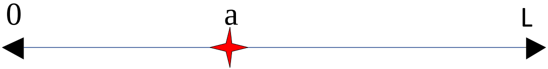
- Pulse-heights:
  - calculate probability of event occurring and energy deposited:
  - (reaching *a* without collision) x (collision at *da*) x (reaching *L* without collision)

$$\int_0^L (e^{-\Sigma a})(\Sigma da)(e^{-\Sigma(L-a)}) = \Sigma(L)e^{-\Sigma(L)}$$

- For *n* collisions followed by an escape, this can be generalized (for $\Sigma = 1$, Detector length = L) to:

$$p_{esc}(n) = \frac{(L)^n e^{-L}}{n!}$$

N Collisions followed by an escape:



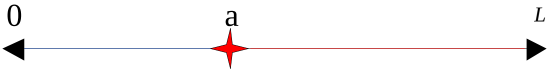
- Volume Flux : calculate (Track length / volume)

$$< T_l >= \frac{\int_0^L a(e^{-\Sigma a})(\Sigma da)(e^{-\Sigma(L-a)})}{\int_0^L (e^{-\Sigma a})(\Sigma da)(e^{-\Sigma(L-a)})}$$

- For *n* collisions followed by an escape, this can be generalized (for $\Sigma = 1$, Detector length = L) to:

$$< T_{l-esc} >= \frac{L}{n+1}$$

$N-1$ Collisions followed by an absorption:



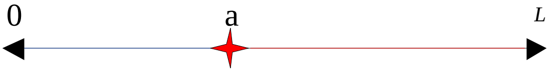
- Pulse-heights:
    - calculate probability of event occurring and energy deposited:
    - (reaching *a* without collision) x (collision at *da*)

$$\int_0^L (e^{-\Sigma a})(\Sigma da) = 1 - \Sigma e^{-\Sigma(L)}$$

- For $n-1$ collisions followed absorption, this can be generalized (for $\Sigma = 1$, Detector length = L) to:

$$p_{abs}(n) = \frac{e^{(-L)}\left[(n-1)!e^{L} - (L)^{n-1} - (n-1)(L)^{n-2} - (n-1)(n-2)(L)^{n-3}\ldots\right]}{(n-1)!}$$

$N - 1$ Collisions followed by an absorption:



- Volume Flux : calculate (Track length / volume)

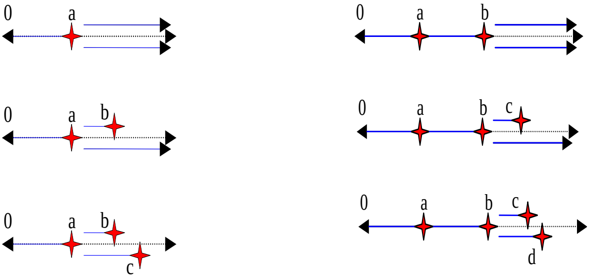
$$< T_l > = \frac{\int_0^L a(e^{-\Sigma a})(\Sigma da)}{\int_0^L (e^{-\Sigma a})(\Sigma da)} = \frac{\Sigma e^{-\Sigma(L)} - L - 1}{\Sigma e^{-\Sigma(L)} - 1}$$

- For $n - 1$ collisions followed absorption, this can be generalized (for $\Sigma = 1$, Detector length = L) to:

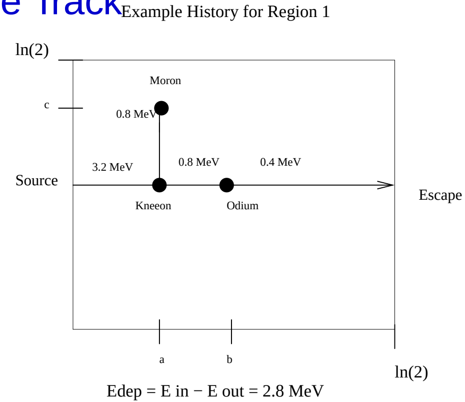
$$T_{l-abs}(n) = \frac{n!e^{(-L)} - L - nL^{n-1} - (n-1)(n)L^{n-2} -}{n!e^{(-L)} - nL^{n-1} - (n-1)(n)L^{n-2} -}$$

Comments on Analytic Solution

- Equations for $p_{esc}(n)$, and $p_{abs}(n)$ account for most interactions.
- “Pair production” collisions below 1 MeV produce two identical particles which require special handling and additional (non-trivial) equations



Example Track



- P = (2 axial collisions) x (1 radial collision)
  - $= (0.5)(0.3)\frac{(ln2)^2 e^{-ln2}}{2}(0.2)(1 - e^{-ln2})$
  - $= 0.0018017$
- 2.8 MeV deposited with probability $= 0.0018017$

Generating high precision analytic results

- Analytic computer code developed[1]
  - Indexing over unique history labeling scheme
  - First region axial leakage source for second region
- For all possible histories
  - Pulse height values calculated
  - Particle currents, leakage and volume fluxes calculated[2]
- Verification of analytic computer code results
  - Selective hand calculation
  - Sum of history probabilities at different tree locations and over all energies and histories

[1] A. Sood (LANL), M.S. Reed (TAMU), R.A. Forster (LANL),"New Results for Pulse-Height Tally Verification" Trans. Amer. Nucl. Soc., 89, 456 (November 2003).

[2] Unpublished

Applying the analytic test problem...

Implementing Analytic Problem:

- Practical problems with probabilistic approach and MCNP
  - Pulse-height tallies are usually for photon/electron problems
  - Physical interactions for photons is hard-coded. Controlling the physics through the data will not work.
  - Very simple code modifications will be required.
- MCNP 5.1.50 calculated results generated for:
  - F1: photon current across interface, last axial surface along diameter for both cylinders
  - F2: photon surface flux across interface, last axial surface along diameter in each cylinder
  - F4: photon volume flux in each cylinder
  - F8: pulse-heights in each cylindrical volumes
  - *F8: total energy deposited in each cylindrical volume

MCNP5.1.50 PHT-VR Verification: Photon Results - analog

Table: **Photon Current, Flux Results**

		Energy (MeV)	Analytic	MCNP 5 (R.E.)	Number of S.D away
photon current	Axial interface		0.8899430000	0.889896 +/- 0	0
	Last axial surface		0.7643510000	0.764251 +/- 0.0001	-1.31
	Diameter				
	Region 1		0.2542390000	0.254223 +/- 0.0002	-0.31
	Region 2		0.1428180000	0.142860 +/- 0.0003	0.98
photon surface flux	Axial interface	0.20	0.0431240772	0.043106 +/- 0.0005	-0.84
		0.40	0.0243696716	0.024377 +/- 0.0005	0.60
		0.80	0.1219679960	0.121936 +/- 0.0002	-1.31
		1.60	0.0688836141	0.068877 +/- 0.0003	-0.30
		3.30	0.3312601617	0.331278 +/- 0.0001	0.54
		total		0.589574 +/- 0	
	Last axial surface				
		0.20	0.0930220670	0.092980 +/- 0.0003	-1.52
		0.40	0.0497322033	0.049750 +/- 0.0004	0.88
		0.80	0.1291299684	0.129123 +/- 0.0002	-0.27
		1.60	0.0688836141	0.068885 +/- 0.0003	0.05
		3.30	0.1656300809	0.165595 +/- 0.0002	-1.06
		total		0.506332 +/- 0.0001	
	Diameter				
	Region 1	0.20	0.0303893949	0.030402 +/- 0.0004	1.07
		0.40	0.0124222561	0.012420 +/- 0.0005	-0.40
		0.80	0.0414075202	0.041392 +/- 0.0003	-1.25
	Region 2	0.20	0.0160899440	0.016097 +/- 0.0006	0.73
		0.40	0.0105163539	0.010522 +/- 0.0006	0.95
		0.80	0.0207037601	0.020705 +/- 0.0004	0.10

MCNP5.1.50 PHT-VR Verification: Photon Results - analog

Table: **Photon Volume Flux Results**

		Energy (MeV)	Analytic	MCNP 5 (R.E.)	Number of S.D away
photon Volume flux	Region 1	0.20	0.0552739100	0.055267 +/- 0.0004	-0.31
		0.40	0.0316929622	0.031707 +/- 0.0004	1.08
		0.80	0.1956661722	0.195609 +/- 0.0002	-1.46
		1.60	0.0439941693	0.043989 +/- 0.0003	-0.43
		3.20	0.4779073926	0.477914 +/- 0.0000	0
		total		+/-	
	Region 2				
		0.20	0.0905357642	0.090521 +/- 0.0003	-0.54
		0.40	0.0618684956	0.061891 +/- 0.0003	1.23
		0.80	0.1903885628	0.190389 +/- 0.0002	0.01
		1.60	0.0716861089	0.071685 +/- 0.0003	-0.05
		3.20	0.2389536963	0.238945 +/- 0.0001	-0.36
		total			

MCNP5.1.50 PHT-VR Verification: Photon Results - analog

Table: **Photon Pulse-Height Results**

		Energy (MeV)	Analytic	MCNP 5 (R.E.)	Number of S.D away
photon pulse height	Region 1		1.1252880000	1.125290 +/- 0.0001	0.02
	Region 2		0.6885630000	0.688757 +/- 0.0001	2.82
	Region 1	0.10	0.5000000000	0.500026 +/- 0.0001	0.52
		1.60	0.1906154747	0.190560 +/- 0.0002	-1.46
		2.00	0.0607580302	0.060770 +/- 0.0004	0.49
		2.20	0.0251175286	0.025103 +/- 0.0006	-0.97
		2.40	0.0685966790	0.068608 +/- 0.0004	0.40
		2.60	0.0100059394	0.009994 +/- 0.0010	-1.19
		2.80	0.0241757434	0.024175 +/- 0.0006	-0.05
		3.00	0.0057451938	0.005749 +/- 0.0013	0.57
		3.20	0.1149854109	0.115015 +/- 0.0003	0.86
		total		1.000000 +/- 0.0000	
	Region 2	0.10	0.4260696364	0.425953 +/- 0.0001	-2.74
		0.20	0.0237685679	0.023765 +/- 0.0006	-0.25
		0.40	0.0630209469	0.063003 +/- 0.0004	-0.71
		0.60	0.0197690310	0.019774 +/- 0.0007	0.34
		0.80	0.0499450801	0.049939 +/- 0.0004	-0.32
		1.00	0.0028098955	0.002813 +/- 0.0019	0.65
		1.20	0.0130114830	0.013023 +/- 0.0009	0.94
		1.40	0.0021885293	0.002190 +/- 0.0021	0.36
		1.60	0.1094651813	0.109498 +/- 0.0003	1.00
		2.00	0.0303790151	0.030382 +/- 0.0006	0.15
		2.20	0.0125587643	0.012574 +/- 0.0009	1.37
		2.40	0.0342983395	0.034307 +/- 0.0005	0.48
		2.60	0.0050029697	0.005002 +/- 0.0014	-0.20
		2.80	0.0120878717	0.012117 +/- 0.0009	2.70
		3.00	0.0028725969	0.002874 +/- 0.0019	0.20
		3.20	0.0574927055	0.057490 +/- 0.0004	-0.12
		total		0.864703 +/- 0.0000	

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- Analytic Verification
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Verification Test Problem

Verification Problem Solution

Pulse Height Results
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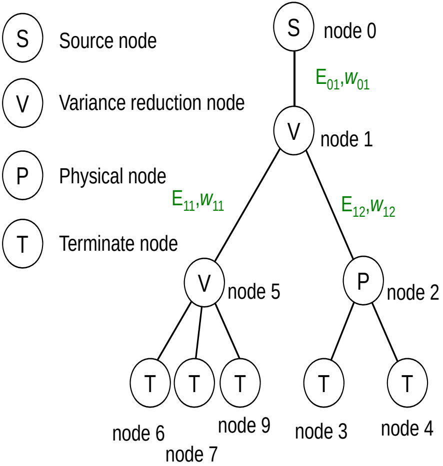
Conclusions

MCNP5.1.50: Pulse-Height Tallies with Variance Reduction

- Pulse height (F8) tallies depend on collections of particles (the entire particle history) and are correlated.
    - Two 1 MeV photons originating from the same event and absorbed in one cell registered as single event at 2 MeV; not two events at 1 MeV counts
    - Assumes both photons have weight one.
    - Not the case when using variance reduction
- Create “trees” to keep track of relationship between individual particles
    - Weight assigned to branches of the tree
    - Energy deposited and weight stored for each branch

Example: PHT with Variance Reduction

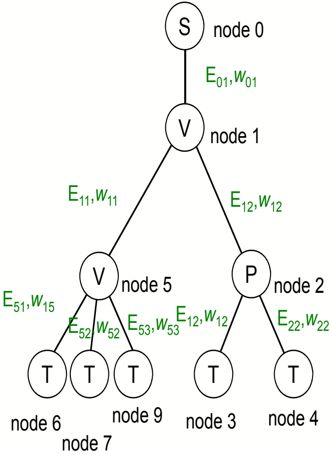


Pulse height tallies require knowledge of the entire particle history

E_{ij} is the energy deposited on branch$_{ij}$.

w_{ij} is the weight multiplier for branch$_{ij}$.

Example: PHT with Variance Reduction



Choice 1
Energy deposited: $E_{01} + E_{11} + E_{51}$
Tree weight: $w_{01}w_{11}w_{51}$

Choice 2
Energy deposited: $E_{01} + E_{11} + E_{52}$
Tree weight: $w_{01}w_{11}w_{52}$

Choice 3
Energy deposited: $E_{01} + E_{11} + E_{53}$
Tree weight: $w_{01}w_{11}w_{53}$

Choice 4
Energy deposited: $E_{01} + E_{12} + E_{21} + E_{21}$
Tree weight: $w_{01}w_{12}w_{21}w_{22}$

Example: PHT with Variance Reduction

- Variance Reduction with Pulse-Height Tallies works for:
    - Geometry / Energy / Time splitting and Roulette
    - Weight Window
    - Exponential Transform
    - Forced Collision
    - DXTRAN
    - Implicit Capture / Weight Cutoff
- Current limitations:
    - Coupled Neutron transport (eg. (n,γ))
    - Electron specific variance reduction
        - Controls types and how many specific types of electrons are produced
        - eg. bremsstrahlung, photon-induced secondary electrons, electron-induced x-rays, knock-on electrons

Verification MCNP5 PHT with Variance Reduction

- Each MCNP VR technique tested; and most combinations

Table: Photon PHT-VR Results Summary

non-integer geom. split / Russ.Roul	WW (generation)
non-integer geom. split / Russ.Roul off	WW usage
Impl Cap. / Wgt Cutoff	WW + Forc. Col.
DXTRAN	WW + Exp. Trans.

- Sequential, MPI/OMP (16 processors, 2 threads each)
- Results are very similar to analog case
  - All results < 2.5 SD away from analytic - normally distributed
  - All results match analytic to 5-6 decimal places

Analytic Problem with ELECTRONS...

- Is it possible to extend this for electron problems?
  - Pulse-height tallies are usually for photon/electron problems
  - Electrons are NOT included in analytic test problem
  - Solution: allow ALL secondary progeny (electrons, brem, xrays, etc)
  - ...but do NOT allow them to escape
    - reflective boundaries for jgp > 1
  - More code modifications required...

MCNP5.1.50 PHT-VR Verification: Photon/Electron Results - analog

Table: **Photon/Electron Surface Current, Flux Results**

		Energy (MeV)	Analytic	MCNP 5 (R.E.)	Number of S.D away
photon current	Axial interface		0.8899430000	0.89000800 +/- 0.0001	0.732
	Last axial surface		0.7643510000	0.76441600 +/- 0.0001	0.852
	Diameter				
	Region 1		0.2542390000	0.25422100 +/- 0.0002	-0.36
	Region 2		0.1428180000	0.14286100 +/- 0.0002	1.502
photon surface flux	Axial interface	0.21	0.0431240772	0.04316740 +/- 0.0005	2.013
		0.41	0.0243696716	0.02435120 +/- 0.0005	-1.53
		0.81	0.1219679960	0.12196600 +/- 0.0002	-0.04
		1.61	0.0688836141	0.06886820 +/- 0.0003	-0.71
		3.30	0.3312601617	0.33129500 +/- 0.0001	1.051
		total		0.58964800 +/- 0.0001	
	Last axial surface				
		0.21	0.0930220670	0.09303850 +/- 0.0002	0.886
		0.41	0.0497322033	0.04973090 +/- 0.0002	-0.11
		0.81	0.1291299684	0.12915800 +/- 0.0002	1.094
		1.61	0.0688836141	0.06886780 +/- 0.0002	-1.13
		3.30	0.1656300809	0.16564500 +/- 0.0002	0.451
		total		0.50644100 +/- 0.0001	
	Diameter				
	Region 1	0.21	0.0303893949	0.03038914 +/- 0.0003	-0.0
		0.41	0.0124222561	0.01242094 +/- 0.0004	-0.2
		0.81	0.0414075202	0.04140325 +/- 0.0002	-0.53
	Region 2	0.21	0.0160899440	0.01609724 +/- 0.0003	1.51
		0.41	0.0105163539	0.01051566 +/- 0.0003	-0.2
		0.81	0.0207037601	0.02071140 +/- 0.0002	1.84

MCNP5.1.50 PHT-VR Verification: Photon/Electron Results - analog

Table: **Photon/Electron Volume Flux Results**

		Energy (MeV)	Analytic	MCNP 5 (R.E.)	Number of S.D away
photon Volume flux	Region 1	0.20	0.0552739100	0.0552899 +/- 0.0003	0.96
		0.40	0.0316929622	0.0316829 +/- 0.0003	-1.06
		0.80	0.1956661722	0.1956360 +/- 0.0001	-1.54
		1.60	0.0439941693	0.0439897 +/- 0.0003	-0.34
		3.20	0.4779073926	0.4779150 +/- 0.0001	0.16
		total		0.8045130 +/- 0.0001	
	Region 2				
		0.20	0.0905357642	0.0905709 +/- 0.0002	1.94
		0.40	0.0618684956	0.0618616 +/- 0.0002	-0.56
		0.80	0.1903885628	0.1904200 +/- 0.0001	1.65
		1.60	0.0716861089	0.0716742 +/- 0.0002	-0.83
		3.20	0.2389536963	0.2389720 +/- 0.0002	0.38
		total		0.6534990 +/- 0.0001	

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Table: **Photon/Electron Pulse-Height Results**

		Energy (MeV)	Analytic	MCNP 5 (R.E.)	Number of S.D away
photon electron pulse height	Region 1		1.1252880000	1.124570 +/- 0.0003	-2.13
	Region 2		0.6885630000	0.688837 +/- 0.0005	0.80
	Region 1	0.10	0.5000000000	0.5001420 +/- 0.0003	0.95
		1.60	0.1906154747	0.1908020 +/- 0.0007	1.40
		2.00	0.0607580302	0.0608153 +/- 0.0012	0.78
		2.20	0.0251175286	0.0251221 +/- 0.002	0.09
		2.40	0.0685966790	0.0685187 +/- 0.0012	-0.95
		2.60	0.0100059394	0.0099869 +/- 0.0031	-0.61
		2.80	0.0241757434	0.0241125 +/- 0.002	-1.31
		3.00	0.0057451938	0.0057073 +/- 0.0042	-1.58
		3.20	0.1149854109	0.1147930 +/- 0.0009	-1.86
		total		1.0000000 +/- 0	
	Region 2	0.10	0.4260696364	0.4260740 +/- 0.0004	0.03
		0.20	0.0237685679	0.0237754 +/- 0.002	0.14
		0.40	0.0630209469	0.0630173 +/- 0.0012	-0.05
		0.60	0.0197690310	0.0196798 +/- 0.0022	-2.06
		0.80	0.0499450801	0.0500267 +/- 0.0014	1.17
		1.00	0.0028098955	0.0028265 +/- 0.0059	1.00
		1.20	0.0130114830	0.0130714 +/- 0.0027	1.70
		1.40	0.0021885293	0.0022040 +/- 0.0067	1.05
		1.60	0.1094651813	0.1096690 +/- 0.0009	2.06
		2.00	0.0303790151	0.0303492 +/- 0.0018	-0.55
		2.20	0.0125587643	0.0125636 +/- 0.0028	0.14
		2.40	0.0342983395	0.0343137 +/- 0.0017	0.26
		2.60	0.0050029697	0.0050024 +/- 0.0045	-0.03
		2.80	0.0120878717	0.0120436 +/- 0.0029	-1.27
		3.00	0.0028725969	0.0028625 +/- 0.0059	-0.60
		3.20	0.0574927055	0.0574907 +/- 0.0013	-0.03
		total		0.8649690 +/- 0.0001	

Verification MCNP5 PHT with Variance Reduction

- Photons physics: Brem, annihilation
- Electrons physics: Compton, PE, Auger, Knock-on
- Each MCNP VR technique tested; and most combinations

Table: Photon PHT-VR Results Summary

non-integer geom. split / Russ.Roul	WW (generation)
non-integer geom. split / Russ.Roul off	WW usage
Impl Cap. / Wgt Cutoff	WW + Forc. Col.
DXTRAN	WW + Exp. Trans.

- Sequential, MPI/OMP (16 processors, 2 threads each)
- Results are very similar to analog case
  - All results < 2.5 SD away from analytic - normally distributed
  - All results match analytic to 5-6 decimal places

Summary: Analytic Verification of PHT-VR in MCNP5.1.50

MCNP5.1.50 pulse-height tallies with VR has been analytically verified

- Calculated analog results for F1, F2, F4, F8, and *F8 tallies compared very well with analytic solution
- Sequential, MPI/OMP 16 processors with 2 threads each
- Virtually all MCNP5.1.50 variance reduction techniques tested...
  - Most combinations of VR tested
  - Energy splitting, time split techniques not tested...
- Results are < 2.5 SD from analytic - normally distributed
- Not all physical events tested...
  - no double fluorescence...
- Can be extended to coincidence/anti-coincidence counting
- Extend problem physics to include backwards scattering

Summary: Probabilistic Approach

- Probabilistic approach to verification worked very well
  - Analytic problem indicated errors in implementation of new feature that would not have been detected easily!
  - Verification of MCNP's PHT-VR work required violating the rules of code verifcation but not the spirit.
- Simplicity of test problem and data does not lower level of verification achieved
  - Data is so controllable, test problems can exercise parts of code that may not be stressed with ordinary test problems (eg (n,2n))
  - Design your own isotopes for your application!
- Can be applied to any transport code, independent of solution technique

Summary: Probabilistic Approach

- Approach can be used to explore other problems
  - Sensitivity studies
    - cross-sections (density, temperature) and boundaries
  - Charged particle transport
  - Production of libraries for multi-group calculations