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Correlated Sampling for Fixed-Source Problems Using MCNP's Tally Fluctuation Chart

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Introduction

- Correlated sampling can be used to reduce the uncertainty of a difference of tallies by taking advantage of the negative covariance term in the sandwich formula; i.e.

$$\text{if } \Delta R = R_1 - R_0, \text{ then } \sigma_{\Delta R}^2 = \sigma_{R_1}^2 + \sigma_{R_0}^2 - 2 \text{cov}(R_1, R_0).$$

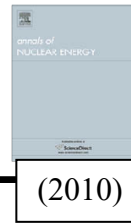
- Correlated sampling can be implemented using MCNP's tally fluctuation chart (TFC):

tally fluctuation charts

nps	mean	tally error	vov	1 slope	fom
64000	5.7968E-01	0.0032	0.0000	4.8	2834429
128000	5.7945E-01	0.0022	0.0000	4.8	2935622
192000	5.7811E-01	0.0018	0.0000	4.8	2975583
256000	5.7753E-01	0.0016	0.0000	10.0	3006336
320000	5.7734E-01	0.0014	0.0000	10.0	3026436
384000	5.7741E-01	0.0013	0.0000	10.0	3050969
448000	5.7787E-01	0.0012	0.0000	10.0	3075052
512000	5.7788E-01	0.0011	0.0000	10.0	3086425
576000	5.7786E-01	0.0011	0.0000	10.0	3095899
640000	5.7799E-01	0.0010	0.0000	10.0	3105174
704000	5.7770E-01	0.0010	0.0000	10.0	3109504
768000	5.7757E-01	0.0009	0.0000	10.0	3114727
832000	5.7766E-01	0.0009	0.0000	10.0	3121714
896000	5.7769E-01	0.0009	0.0000	10.0	3128365
960000	5.7778E-01	0.0008	0.0000	10.0	3134761
1000000	5.7774E-01	0.0008	0.0000	10.0	3135895

- In this talk, I will:
 1. Discuss how to do this.
 2. Discuss how to do this for other combinations of tallies.
 3. Present a general-purpose code that I wrote to do this.
 4. Present results.

Previous Work



(2010)

On using correlated sampling to simulate small changes in system response by MCNP

Tao He, Bingjing Su*

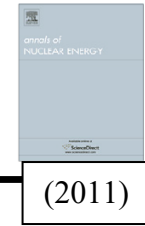
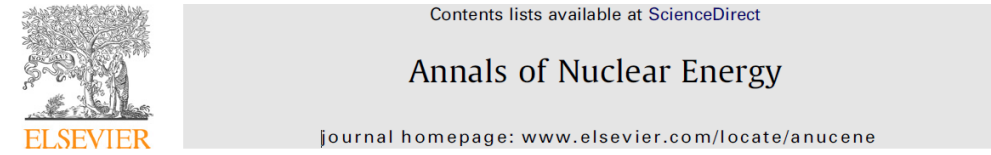
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ABSTRACT

Correlated sampling is a developed technique that enables MCNP to simulate small changes in system response with an acceptable uncertainty at reasonable number of particle histories. In this work the performance of correlated sampling is investigated. It is found that under certain conditions using the default output format of MCNP and the original practice of batch statistics developed for correlated sampling may overestimate the uncertainty of a response change. The cause of the overestimation is analyzed; correspondingly two suggestions are proposed for correlated sampling to ensure correct estimation of the uncertainty. The performance of improved correlated sampling is compared with that of un-correlated sampling computation to demonstrate the effect of correlated sampling. Results show that correlated sampling indeed is a robust technique for MCNP to simulate small changes in system response.



(2011)

Comparison between correlated sampling and the perturbation technique of MCNP5 for fixed-source problems

Tao He, Bingjing Su*

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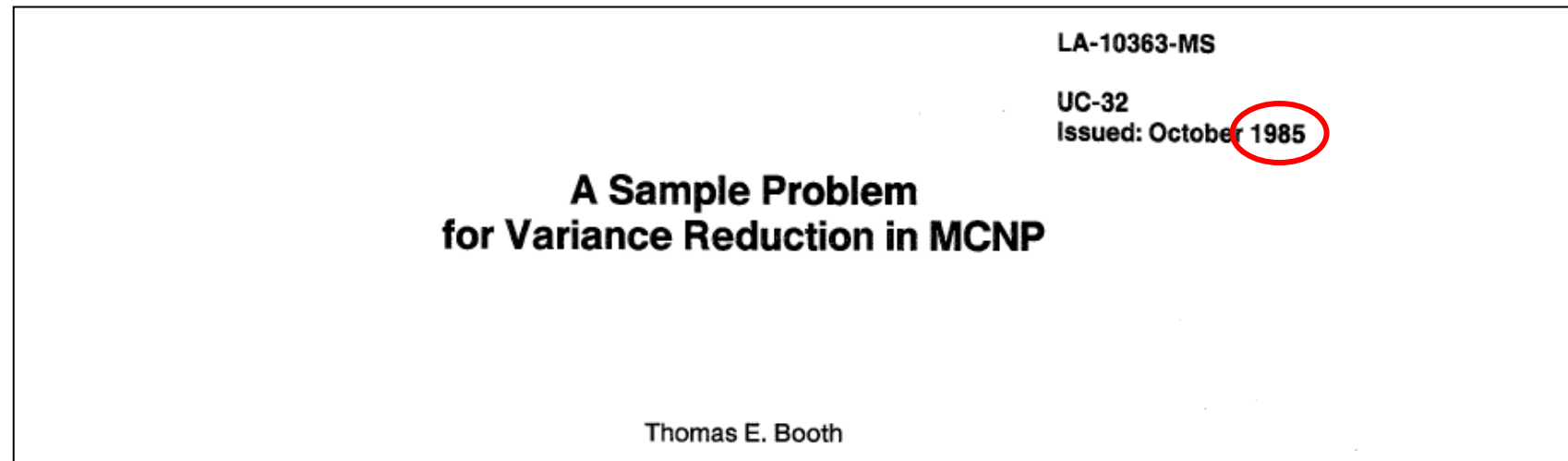
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ABSTRACT

Correlated sampling and the differential operator perturbation technique are two methods that enable MCNP (Monte Carlo N-Particle) to simulate small response change between an original system and a perturbed system. In this work the performance of the MCNP differential operator perturbation technique is compared with that of the MCNP correlated sampling method for three types of fixed-source problems. In terms of precision of predicted response changes, the MCNP perturbation technique outperforms correlated sampling for the problem involving variation of nuclide concentrations in the same direction but performs comparably with or even underperforms correlated sampling for the other two types of problems that involve void or variation of nuclide concentrations in opposite directions. In terms of accuracy, the MCNP differential operator perturbation calculations may predict inaccurate results that deviate from the benchmarks well beyond their uncertainty ranges for some of the test problems. However, the accuracy of the MCNP differential operator perturbation can be improved if the midpoint correction technique is used.



Booth's Explanation

- T. E. Booth, LA-10363-MS (1985).

- In other words, the score for batch i is

$$t_i = \frac{M_i N_i - M_{i-1} N_{i-1}}{N_i - N_{i-1}}, i = 1, \dots, N; M_0 = N_0 = 0$$

- The difference is
 $(5.85115E-20 - 5.90955E-20) = 5.84000E-20$.

- The uncorrelated uncertainty is
 $7.435E-21 = 1273\%$.

- The correlated uncertainty is 8%!

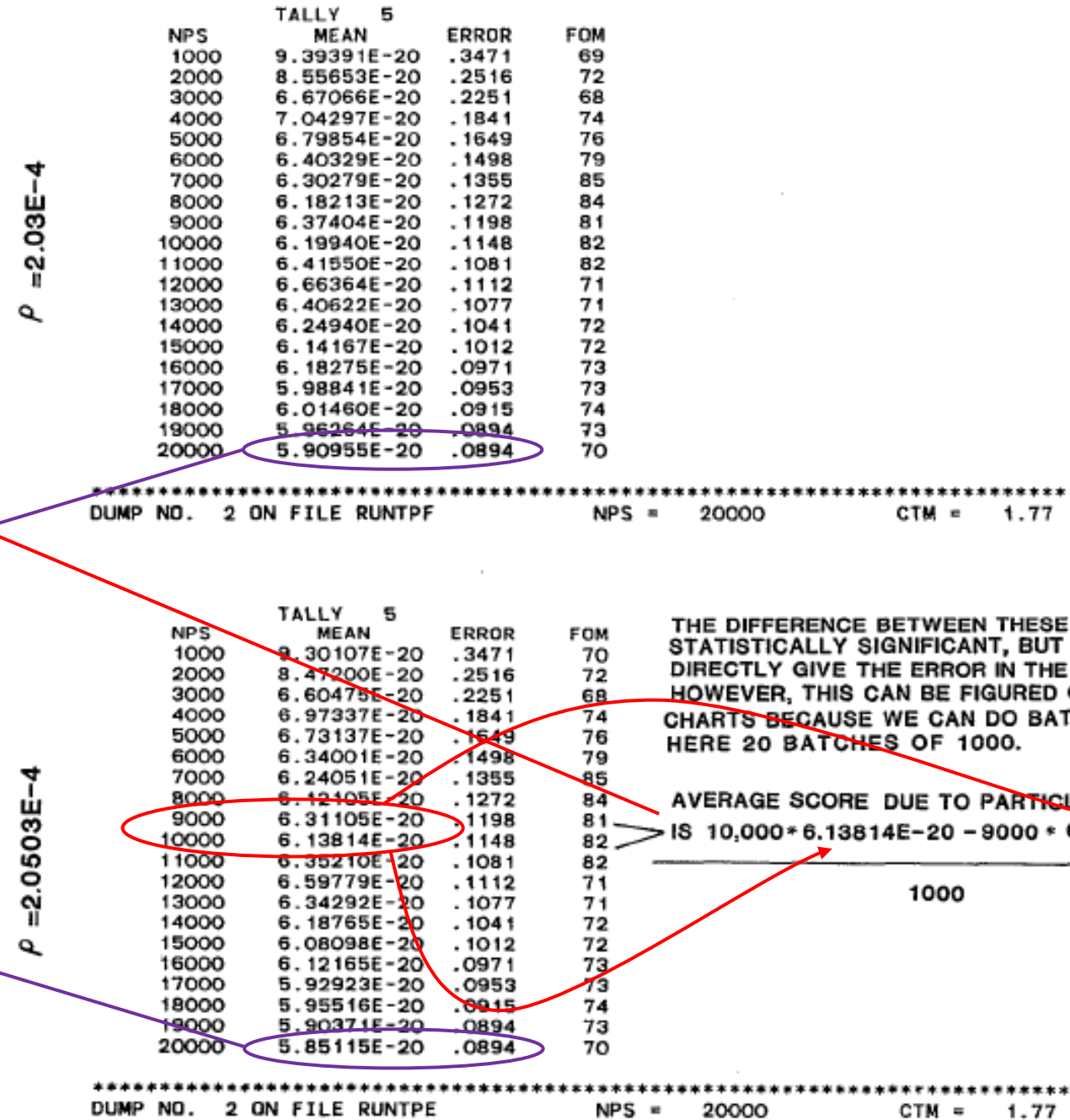


FIG. 46. PROCESS FOR OBTAINING BATCH STATISTICS.

He and Su's Explanation

- T. He and B. Su, *Ann. Nucl. Energy*, **37**, 34–42 (2010).

- In other words, the variance

$$\sigma_{\Delta R}^2 = \sigma_{R_1}^2 + \sigma_{R_0}^2 - 2 \text{cov}(R_1, R_0)$$

is estimated from N batches using

$$\sigma_{\Delta R}^2 = \frac{1}{N(N-1)} \sum_{i=1}^N [(f_i - R_1)^2 + (g_i - R_0)^2] - \frac{2}{N(N-1)} \sum_{i=1}^N (f_i - R_1)(g_i - R_0),$$

where R_1 and R_0 are the final tallies.

$$\Delta I = I_1 - I_2 = \frac{1}{n} \sum_{i=1}^n (f_i - g_i). \quad (1)$$

The variance of ΔI is given by

$$\begin{aligned} \sigma_{\Delta I}^2 &= \frac{1}{n(n-1)} \cdot \sum_{i=1}^n [(f_i - g_i) - (I_1 - I_2)]^2 \\ &= \sigma_1^2 + \sigma_2^2 - 2 \text{cov}(I_1, I_2), \end{aligned} \quad (2)$$

where $\text{cov}(I_1, I_2)$ is the covariance between the two processes, defined by:

$$\text{cov}(I_1, I_2) = \frac{1}{n(n-1)} \cdot \sum_{i=1}^n [(f_i - I_1)(g_i - I_2)]. \quad (3)$$

The quantity n in the above equations represents the total number of particle histories, i.e., NPS. Note that when f_i and g_i are statistically independent as in the cases of un-correlated MCNP simulations, the covariance given in Eq. (3) is zero and thus the variance of ΔI is reduced to

$$\sigma_{\Delta I}^2 = \sigma_1^2 + \sigma_2^2. \quad (4)$$

He and Su Test Problem

- Sphere of coal with radius 30 cm and density 0.67552 g/cm³.
- Point source of neutrons with a uniform energy distribution from 1 to 14.1 MeV.
- The source strength is 1 neutron/s.
- The response of interest is the total neutron leakage.
- The coal density is perturbed by +1%.
- Analysis of 1000 realizations of correlated and uncorrelated pairs using 1000 batches of 1000 histories each (no implicit capture).

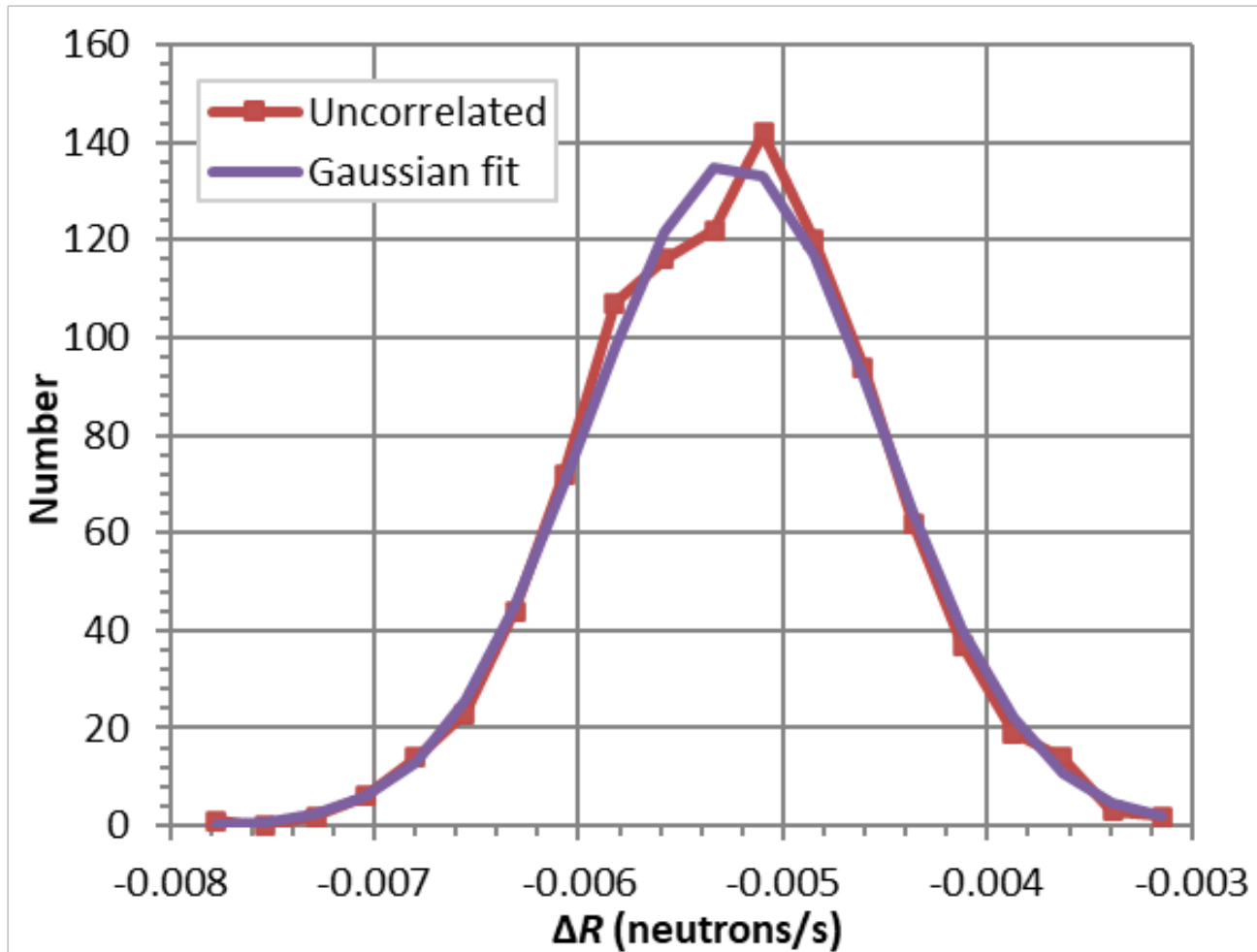
Correlated?	Uncorrelated	Correlated
Average $\sigma_{\Delta R}$ (/s)	6.9897E-04	7.2469E-05
Theoretical $\sigma_{\Delta R}$ (/s)	6.9922E-04	7.2390E-05
Difference	-0.04%	0.11%
s_p (/s)	7.0739E-04	7.2726E-05
Difference	-1.19%	-0.35%
Gaussian Fit (/s)	7.1841E-04	7.5836E-05
Difference	-2.71%	-4.44%

$$s_p = \sqrt{\frac{1}{(N-1)} \sum_{i=1}^N (P_i - \bar{P})^2}$$

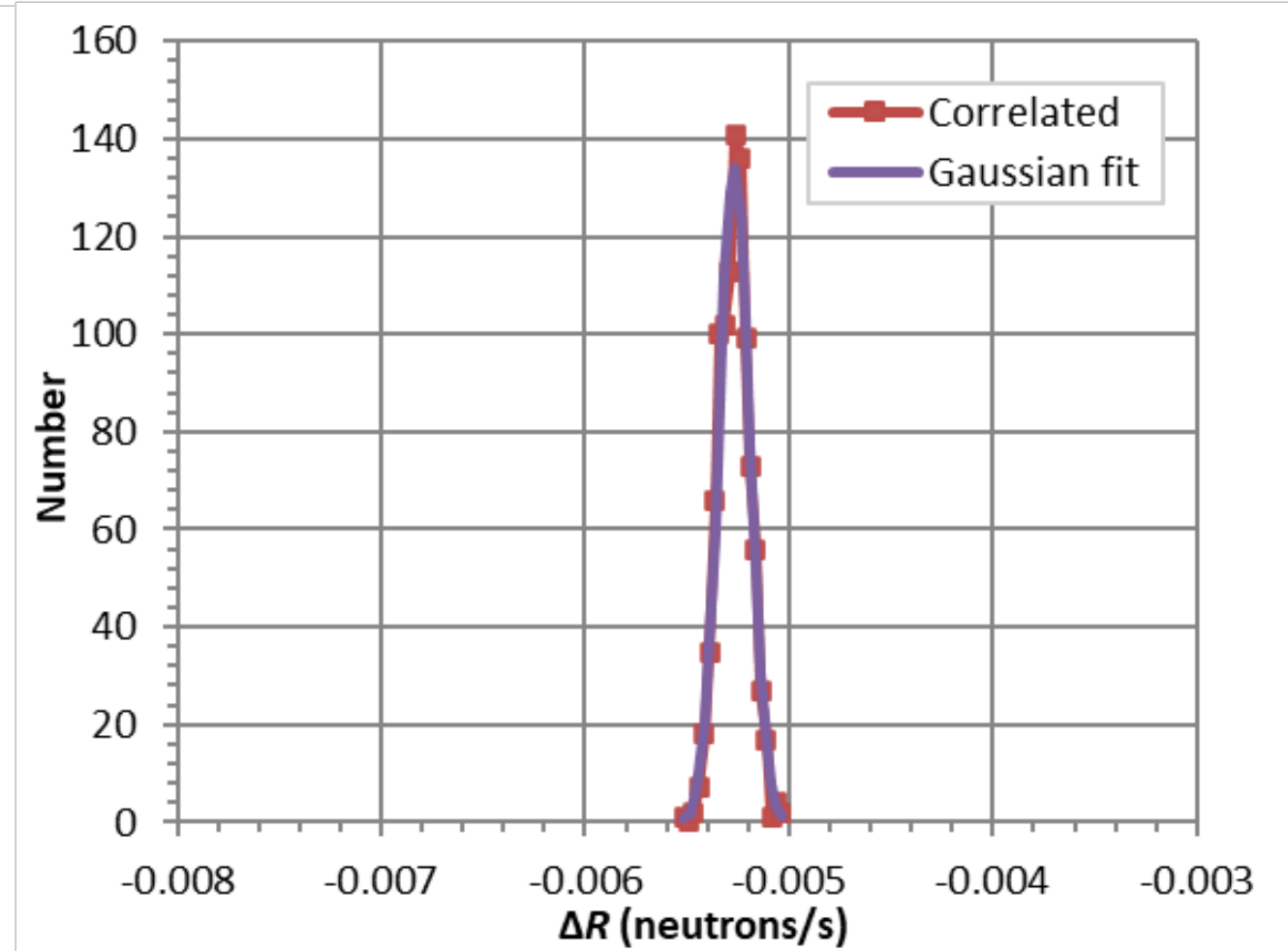
- Correlated sampling reduces the uncertainty of the difference by a factor of 9.5 for the same number of histories!

He and Su Test Problem—Gaussian Fits for ΔR

- Histogram of ΔR (leakage) for 1000 pairs of realizations with no variance reduction.



(a) Uncorrelated



(b) Correlated

Correlated Sampling for a Ratio of Random Variables

- The ratio of two Monte Carlo tallies is $R_R = \frac{R_1}{R_0}$.

- If the tallies are independent, the ratio has variance

$$\sigma_{R_R}^2 = R_R^2 \left[\left(\frac{\sigma_{R_1}}{R_1} \right)^2 + \left(\frac{\sigma_{R_0}}{R_0} \right)^2 \right].$$

- If the tallies are correlated, the variance is approximately

$$\sigma_{R_R}^2 \approx R_R^2 \left[\left(\frac{\sigma_{R_1}}{R_1} \right)^2 + \left(\frac{\sigma_{R_0}}{R_0} \right)^2 - 2 \frac{\text{cov}(R_1, R_0)}{R_1 R_0} \right].$$

- The variance can be estimated from N correlated pairs of samples (f_i, g_i) using

$$\sigma_{R_R}^2 \approx \frac{R_R^2}{N(N-1)} \sum_{i=1}^N \left[\left(\frac{f_i - R_1}{R_1} \right)^2 + \left(\frac{g_i - R_0}{R_0} \right)^2 \right] - \frac{2R_R^2}{N(N-1)} \sum_{i=1}^N \frac{(f_i - R_1)(g_i - R_0)}{R_1 R_0}.$$

Why Are Ratios Important?

- The first-order relative sensitivity $S_{R,x}$ of a response R with respect to input parameter x is

$$S_{R,x} \equiv \frac{x_0}{R(x_0)} \frac{\partial R}{\partial x} \Big|_{x=x_0}.$$

- The differential operator (Taylor series) perturbation method in MCNP (the PERT card) can be used to calculate $S_{R,x}$ as

$$S_{R,x} = c_1 \frac{R_1}{R_0}.$$

- Another way to calculate $S_{R,x}$ is to use a forward-difference estimate of the derivative, yielding the approximation

$$S_{R,x} \approx \frac{x_0}{R(x_0)} \frac{R(x_0 + \Delta x) - R(x_0)}{(x_0 + \Delta x) - x_0} = c_1 \left(\frac{R_+}{R_0} - 1 \right),$$

or a backward-difference estimate of the derivative, yielding the approximation

$$S_{R,x} \approx \frac{x_0}{R(x_0)} \frac{R(x_0) - R(x_0 - \Delta x)}{x_0 - (x_0 - \Delta x)} = c_1 \left(1 - \frac{R_-}{R_0} \right).$$

Correlated Sampling for the Difference of Ratios of Random Variables

- Using a central-difference estimate of the derivative yields the approximation

$$S_{R,x} \approx \frac{x_0}{R(x_0)} \frac{R(x_0 + \Delta x) - R(x_0 - \Delta x)}{(x_0 + \Delta x) - (x_0 - \Delta x)} = c_2 \frac{R_+ - R_-}{R_0}.$$

- If the tallies are independent, the central difference has variance

$$\sigma_{S_{R,x}}^2 = \left(\frac{c_2}{R_0} \right)^2 (\sigma_{R_+}^2 + \sigma_{R_-}^2) + \left(\frac{S_{R,x}}{R_0} \right)^2 \sigma_{R_0}^2.$$

- If the tallies are correlated, the variance is approximately

$$\begin{aligned} \sigma_{S_{R,x}}^2 &\approx \left(\frac{c_2}{R_0} \right)^2 (\sigma_{R_+}^2 + \sigma_{R_-}^2 - 2 \text{cov}(R_+, R_-)) + \left(\frac{S_{R,x}}{R_0} \right)^2 \sigma_{R_0}^2 \\ &\quad - \frac{2c_2 S_{R,x}}{R_0^2} (\text{cov}(R_+, R_0) - \text{cov}(R_-, R_0)). \end{aligned}$$

- The variance can be estimated from N correlated triplets of samples (f_i, g_i, h_i) using

$$\begin{aligned} \sigma_{S_{R,x}}^2 &\approx \left(\frac{c_2}{R_0} \right)^2 \frac{1}{N(N-1)} \sum_{i=1}^N [(f_i - R_+)^2 + (g_i - R_-)^2] - 2 \left(\frac{c_2}{R_0} \right)^2 \frac{1}{N(N-1)} \sum_{i=1}^N (f_i - R_+)(g_i - R_-) \\ &\quad + \left(\frac{S_{R,x}}{R_0} \right)^2 \frac{1}{N(N-1)} \sum_{i=1}^N (h_i - R_0)^2 - \frac{2c_2 S_{R,x}}{R_0^2} \frac{1}{N(N-1)} \sum_{i=1}^N [(f_i - R_+) - (g_i - R_-)](h_i - R_0). \end{aligned}$$

This Requires More Digits in the TFC and More Entries in the TFC

- He and Su (2010) discovered that inaccuracies sometimes build up with the default TFC format.
 - + Affects TFCs with many batches and small batch size.
- Setting the 50th entry on the “debug information” card DBCN to 2 changes

1tally fluctuation charts

nps	mean	tally error	vov	1 slope	fom
64000	5.7968E-01	0.0032	0.0000	4.8	2834429
128000	5.7945E-01	0.0022	0.0000	4.8	2935622
192000	5.7811E-01	0.0018	0.0000	4.8	2975583
256000	5.7753E-01	0.0016	0.0000	10.0	3006336
320000	5.7734E-01	0.0014	0.0000	10.0	3026436
384000	5.7741E-01	0.0013	0.0000	10.0	3050969
448000	5.7787E-01	0.0012	0.0000	10.0	3075052
512000	5.7788E-01	0.0011	0.0000	10.0	3086425
576000	5.7786E-01	0.0011	0.0000	10.0	3095899
640000	5.7799E-01	0.0010	0.0000	10.0	3105174
704000	5.7770E-01	0.0010	0.0000	10.0	3109504
768000	5.7757E-01	0.0009	0.0000	10.0	3114727
832000	5.7766E-01	0.0009	0.0000	10.0	3121714
896000	5.7769E-01	0.0009	0.0000	10.0	3128365
960000	5.7778E-01	0.0008	0.0000	10.0	3134761
1000000	5.7774E-01	0.0008	0.0000	10.0	3135895

to

1tally fluctuation charts

nps	mean	tally error	vov	1 slope	fom
64000	5.79679864E-01	0.00318	3.5E-06	4.8	2747001
128000	5.79449736E-01	0.00225	1.7E-06	4.8	2880201
192000	5.78112773E-01	0.00184	1.1E-06	4.8	2932709
256000	5.77531802E-01	0.00160	8.4E-07	10.0	2970167
320000	5.77339425E-01	0.00143	6.7E-07	10.0	2994195
384000	5.77412159E-01	0.00130	5.6E-07	10.0	3020949
448000	5.77868992E-01	0.00121	4.8E-07	10.0	3046648
512000	5.77877612E-01	0.00113	4.2E-07	10.0	3059574
576000	5.77862406E-01	0.00106	3.8E-07	10.0	3070100
640000	5.77993026E-01	0.00101	3.4E-07	10.0	3080471
704000	5.77701281E-01	0.00096	3.1E-07	10.0	3085518
768000	5.77567547E-01	0.00092	2.8E-07	10.0	3091152
832000	5.77664540E-01	0.00088	2.6E-07	10.0	3098207
896000	5.77693541E-01	0.00085	2.4E-07	10.0	3105079
960000	5.77780383E-01	0.00082	2.3E-07	10.0	3111693
1000000	5.77740591E-01	0.00081	2.2E-07	10.0	3112974

- How to get more than 20 entries in the TFC?
 - + Modify the MCNP6 source code.
 - + Use more input files.

How Do You Do This?

- Remember this is for fixed-source problems only.
- Run your base case with
 - + “DBCN 49j 2” to get more digits in the TFC.
 - + “RAND GEN=2 SEED=*seed* HIST=*hist*” to control the random number seed and history number (and improve the random number generator).
- If using my version of MCNP, use

```
NPS nps1
PRDMP 4j nps1/1000 1000
RAND GEN=2 SEED=seed
```
- If using the standard installation of MCNP, then you need five realizations *n* of

```
NPS nps2
PRDMP 4j nps2/20
RAND GEN=2 SEED=seed HIST=(n-1)×nps2+1
```

where $nps_2 = nps_1/5$ and the output files are named the same except with 1–5 (and any number of leading zeros) somewhere.
- Run positive and/or negative perturbed cases the same way (and if using standard MCNP, then with the same naming convention).

COSUBS—COrelated Sampling Using Batch Statistics

- Michael Squire, U. Texas-San Antonio, wrote an early Python version of this code.

Flag	Description	Default
-b	MCNP output file for base case (required except with -h or -v options)	None
-n	MCNP output file for negative perturbation	None
-p	MCNP output file for positive perturbation	None
-r	Relative perturbation used in MCNP calculations	None
-f	Tally to divide other tallies by	None
-u	abs / rel / prc Output absolute/relative/percent uncertainties	abs
-o	Output file to write	None
-v	Print code version and exit	None
-h	Print command-line options and exit	None

- The MCNP output files must be from identical inputs except for some perturbation.
- There must be a base (unperturbed) MCNP output file.
- If an MCNP output file name includes a string of #'s, they are assumed to represent a sequential series of numbers starting at 1: e.g. “###” represents 001 to 999 (they need not all be present but those present must be sequential).

COSUBS Options and Logic

- If there is a relative perturbation specified (-r), then differences and relative sensitivities are output. If not, then only differences, ratios, or differences of ratios are printed.
- If there is a base MCNP output file and a positive-perturbation MCNP output file (-p), differences are (positive – base) and sensitivities are (positive – base)/base.
- If there is a MCNP base output file and a negative-perturbation MCNP output file (-n), differences are (negative – base) but sensitivities are (base – negative)/base.
- If there is a base MCNP output file, a negative-perturbation output file (-n), and a positive-perturbation output file (-p), differences are (positive – negative) and sensitivities are (positive – negative)/base.
- If there is a tally specified (-f), then differences are differences of ratios and sensitivities are sensitivities of ratios, with the specified tally as the denominator.
- If there is no negative- or positive-perturbation output file, the base output file and command line must include either
 - (1) perturbations (PERT cards) and a relative perturbation (-r), or
 - (2) a normalization tally (-f).
 - + If there are perturbations, METHOD = 2 is assumed and the tally perturbations are printed as the difference.
 - + If there is a normalization tally (-f), the ratio is printed instead of the difference.

Ouput for the He-Su Coal Sphere—Difference

```
sn.compile% cosubs -b HeSu0_s0001_1e6.o -p HeSulp_s0001_1e6.o
```

```
COrrrelated SAmpling with Batch Statistics using MCNP TFCs, version 20230913
```

```
command line: cosubs -b HeSu0_s0001_1e6.o -p HeSulp_s0001_1e6.o
```

```
base file: HeSu0_s0001_1e6.o
```

```
negative file: <none>
```

```
positive file: HeSulp_s0001_1e6.o
```

```
relative pert: 0.0000E+00
```

```
tally for norm: <none>
```

```
unc. output: abs
```

```
output file: <none>
```

```
number of files: 1
```

```
number of perturbations: 0
```

```
number of batches: 1000
```

```
reading HeSu0_s0001_1e6.o...
```

```
reading HeSulp_s0001_1e6.o...
```

```
number of histories: 1000000
```

```
uncertainties are absolute, not relative.
```

```
tally 1
```

	Difference		Uncertainty	
B	Difference	Cov,batch	NoCov,batch	NoCov,nobch
10	-5.191757E-03	4.513918E-05	4.800015E-04	6.625296E-04
20	-5.191757E-03	4.632082E-05	6.036956E-04	6.625296E-04
25	-5.191757E-03	5.215030E-05	6.168289E-04	6.625296E-04
40	-5.191757E-03	5.800320E-05	5.780000E-04	6.625296E-04
50	-5.191757E-03	6.193640E-05	6.199132E-04	6.625296E-04
100	-5.191757E-03	6.242216E-05	5.997531E-04	6.625296E-04
125	-5.191757E-03	6.282734E-05	6.217696E-04	6.625296E-04
200	-5.191757E-03	6.748244E-05	6.372751E-04	6.625296E-04
250	-5.191757E-03	6.464526E-05	6.434108E-04	6.625296E-04
500	-5.191757E-03	6.524566E-05	6.785121E-04	6.625296E-04
1000	-5.191757E-03	6.917412E-05	6.630635E-04	6.625296E-04

Ouput for the He-Su Coal Sphere—Central Difference Estimate of Relative Sensitivity

```
sn.compile% cosubs -b HeSu0_s0001_1e6.o -p HeSulp_s0001_1e6.o -n HeSulm_s0001_1e6.o -r .01
```

COrrrelated SAMpling with Batch Statistics using MCNP TFCs, version 20230913

```
command line: cosubs -b HeSu0_s0001_1e6.o -p HeSulp_s0001_1e6.o -n HeSulm_s0001_1e6.o -r .01
```

```
base file: HeSu0_s0001_1e6.o
```

```
negative file: HeSulm_s0001_1e6.o
```

```
positive file: HeSulp_s0001_1e6.o
```

```
relative pert: 1.0000E-02
```

```
tally for norm: <none>
```

```
unc. output: abs
```

```
output file: <none>
```

```
number of files: 1
```

```
number of perturbations: 0
```

```
number of batches: 1000
```

```
reading HeSu0_s0001_1e6.o...
```

```
reading HeSulp_s0001_1e6.o...
```

```
reading HeSulm_s0001_1e6.o...
```

```
number of histories: 1000000
```

```
uncertainties are absolute, not relative.
```

```
tally 1
```

	Difference	Cov, batch	Uncertainty NoCov, batch	Uncertainty NoCov, nobch	Rel.Sens. Rel.Sens.	Cov, batch	Uncertainty NoCov, batch	Uncertainty NoCov, nobch
B	Difference	Cov, batch	NoCov, batch	NoCov, nobch	Rel.Sens.	Cov, batch	NoCov, batch	NoCov, nobch
10	-1.050004E-02	7.739781E-05	4.970771E-04	6.614520E-04	-9.092012E-01	6.670237E-03	4.304524E-02	5.728004E-02
20	-1.050004E-02	7.938582E-05	6.129421E-04	6.614520E-04	-9.092012E-01	6.884069E-03	5.307899E-02	5.728004E-02
[...]								
100	-1.050004E-02	9.870230E-05	6.056830E-04	6.614520E-04	-9.092012E-01	8.560759E-03	5.245044E-02	5.728004E-02
125	-1.050004E-02	9.314113E-05	6.216888E-04	6.614520E-04	-9.092012E-01	8.131045E-03	5.383660E-02	5.728004E-02
200	-1.050004E-02	9.979212E-05	6.417886E-04	6.614520E-04	-9.092012E-01	8.649243E-03	5.557713E-02	5.728004E-02
250	-1.050004E-02	9.520013E-05	6.450581E-04	6.614520E-04	-9.092012E-01	8.291318E-03	5.586029E-02	5.728004E-02
500	-1.050004E-02	9.421104E-05	6.779814E-04	6.614520E-04	-9.092012E-01	8.222887E-03	5.871142E-02	5.728004E-02
1000	-1.050004E-02	9.621574E-05	6.615314E-04	6.614520E-04	-9.092012E-01	8.379961E-03	5.728694E-02	5.728004E-02

He and Su Test Problem

- Sphere of coal with radius 30 cm and density 0.67552 g/cm³.
- Point source of neutrons with a uniform energy distribution from 1 to 14.1 MeV.
- The source strength is 1 neutron/s.
- The response of interest is the total neutron leakage.
- The coal density is perturbed by ±1% in order to estimate the relative sensitivity.
- Analysis of 1000 realizations of correlated and uncorrelated triples using 1000 batches of 1000 histories each (no implicit capture).

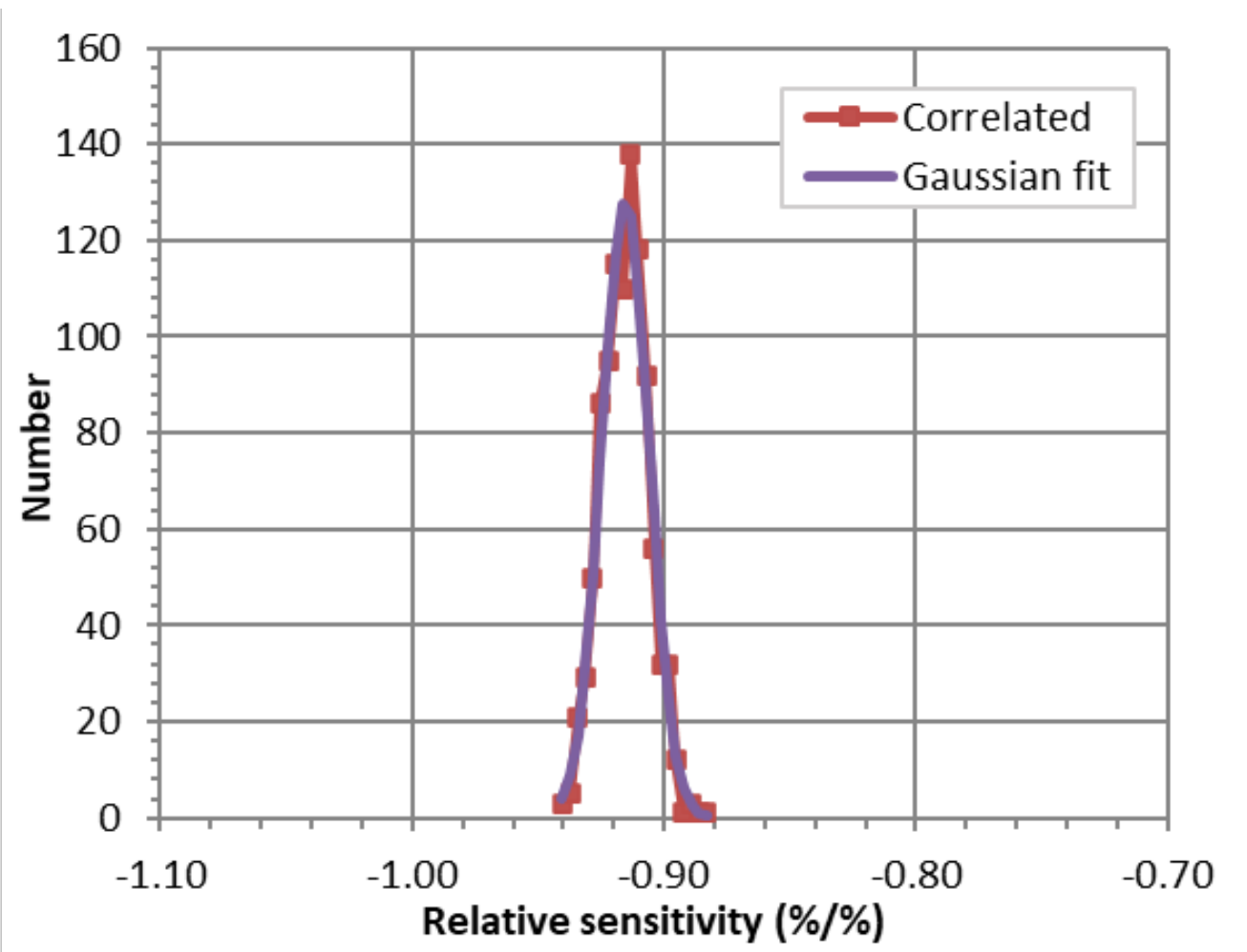
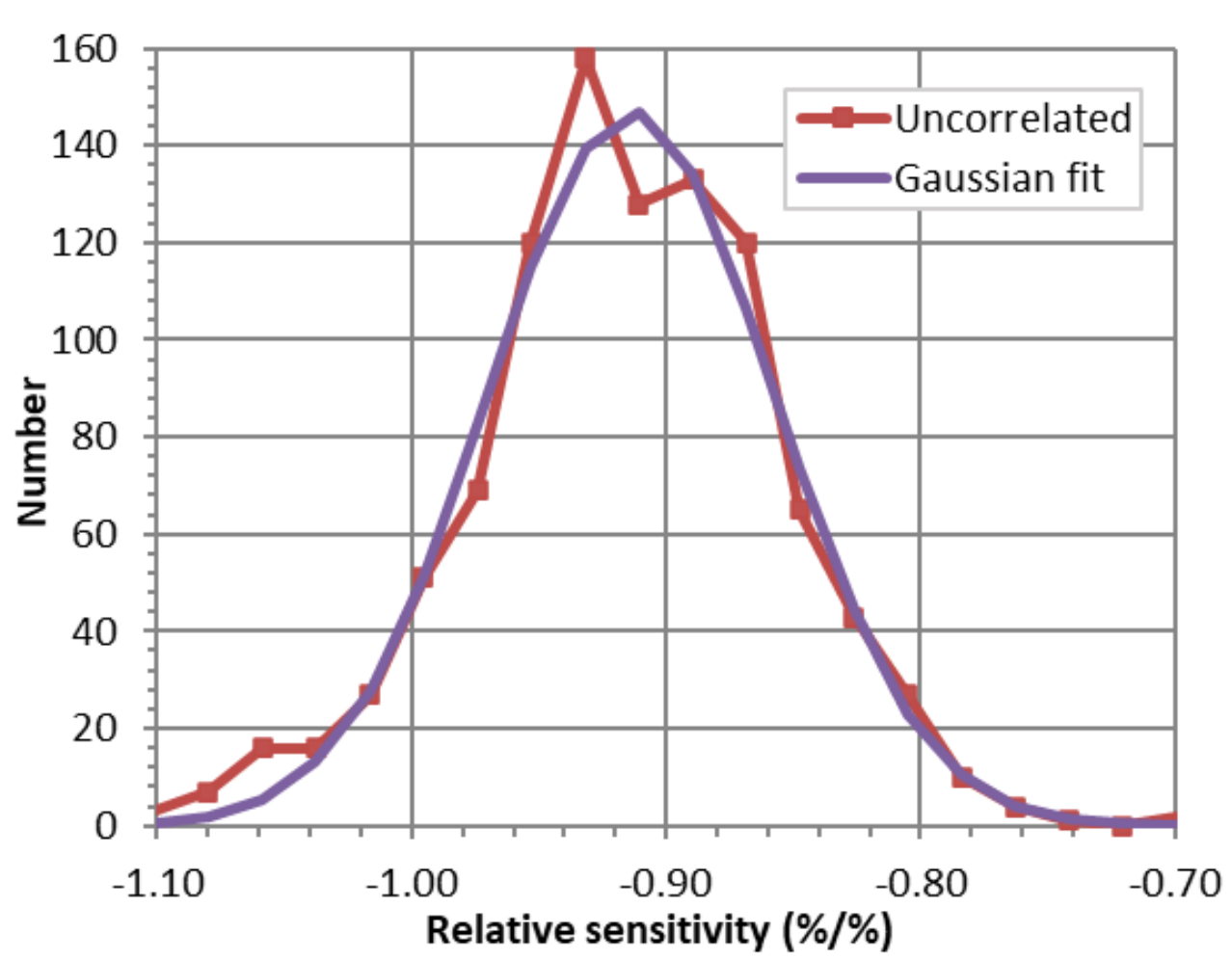
Correlated?	Uncorrelated	Correlated
Average $\sigma_{S,x}$ (/s)	6.0540E-02	8.9104E-03
Theoretical $\sigma_{S,x}$ (/s)	6.0538E-02	8.9061E-03
Difference	0.00%	0.05%
s_p (/s)	6.0107E-02	9.1660E-03
Difference	0.72%	-2.79%
Gaussian Fit (/s)	5.6379E-02	9.4587E-03
Difference	7.38%	-5.80%

$$s_p = \sqrt{\frac{1}{(N-1)} \sum_{i=1}^N (P_i - \bar{P})^2}$$

- Correlated sampling reduces the uncertainty of the sensitivity by a factor of 6.7 for the same number of histories!

He and Su Test Problem—Gaussian Fits for S

- Histogram of the relative sensitivity of the leakage with respect to the coal density for 1000 triplets of realizations with no variance reduction.



(a) Uncorrelated
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NATIONAL LABORATORY
EST. 1943

(b) Correlated

Košťál Test Problem

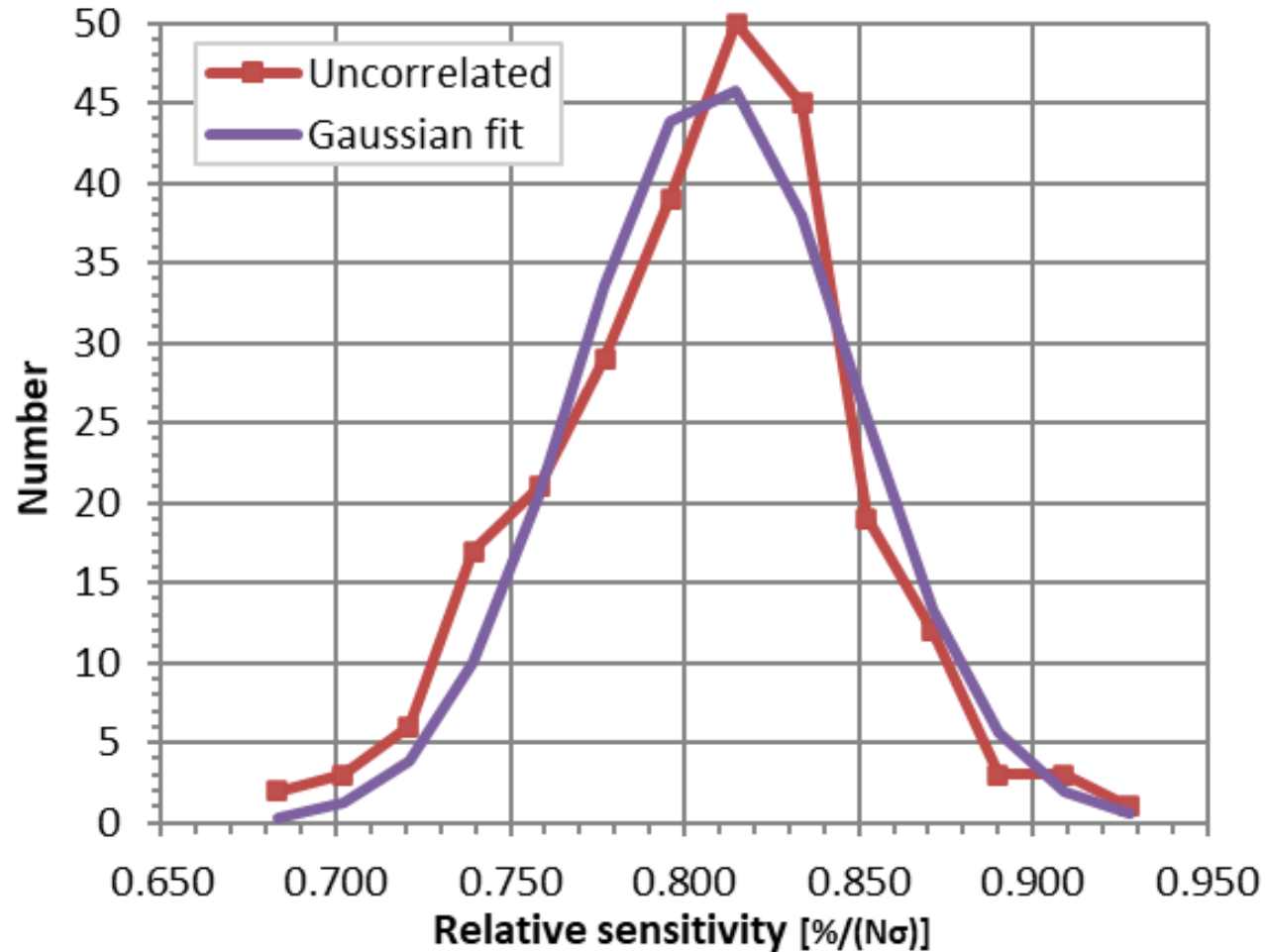
- A stainless steel block of height and width 50.2 cm; thickness 50.4 cm; and density 7.9083 g/cm³.
- A ²⁵²Cf neutron source is in the geometric center.
- The source strength is 1 neutron/s.
- The response of interest is the spectrum 100 cm from the source—specifically, the flux in $0.5623413 < E \leq 0.5956621$ MeV.
- The block height is perturbed by $\pm 3\sigma$ in order to estimate the relative sensitivity using two one-sided differences.
- Analysis of 250 realizations of correlated and uncorrelated pairs using 1000 batches of 1000 histories each (no implicit capture).

Correlated?	Backward Difference		Forward Difference	
	Uncorrelated	Correlated	Uncorrelated	Correlated
Average $\sigma_{S,x}$ (/s)	4.6217E-02	1.1755E-02	4.7969E-02	1.1504E-02
s_p (/s)	4.2402E-02	1.1763E-02	4.7031E-02	1.1962E-02
Difference	9.00%	-0.07%	2.00%	-3.83%
Gaussian Fit (/s)	3.9554E-02	1.1983E-02	4.7747E-02	1.2379E-02
Difference	16.84%	-1.90%	0.46%	-7.07%

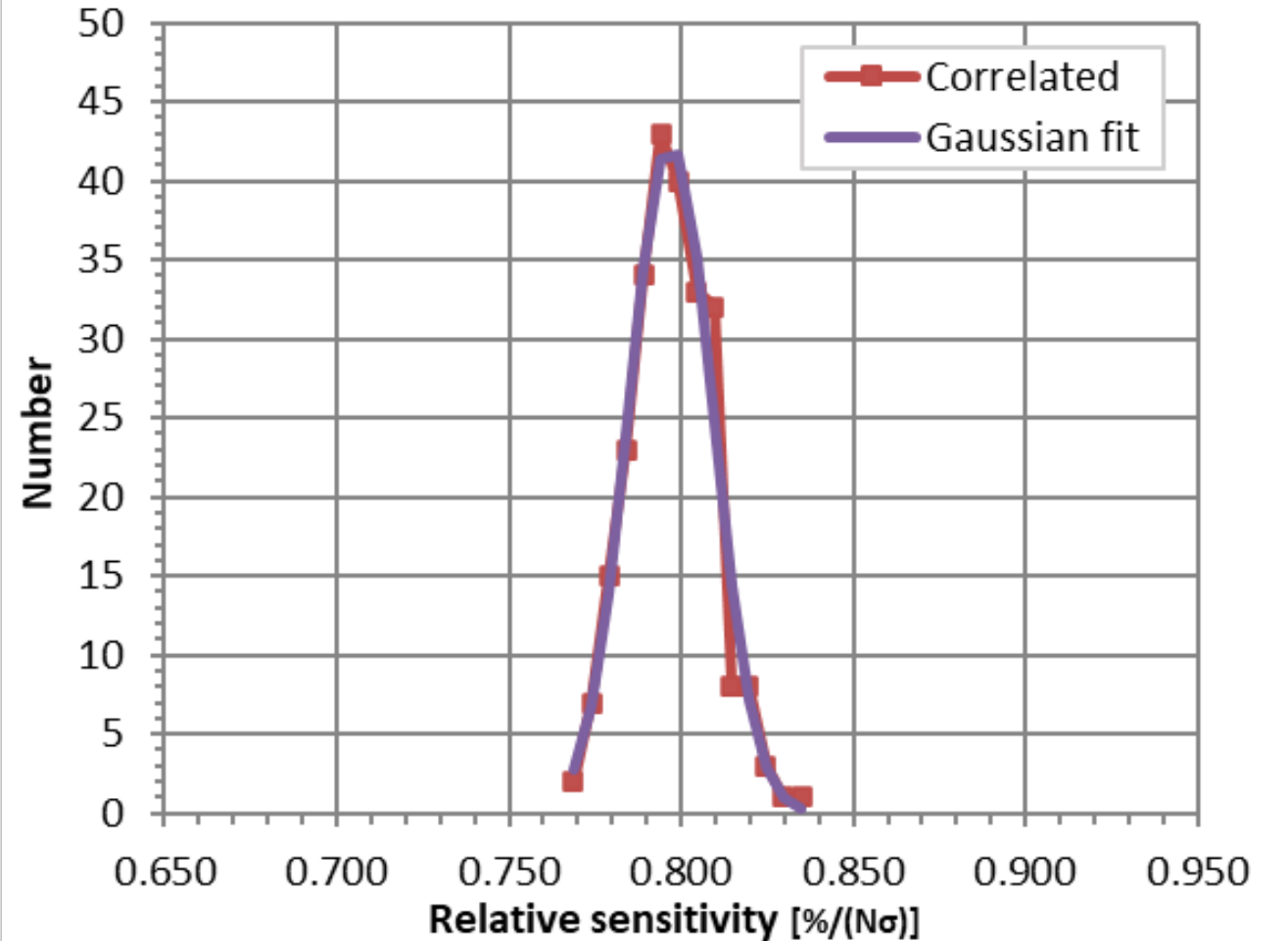
- Correlated sampling reduces the uncertainty of the sensitivity by a factor of 4 for the same number of histories!

Koš'ál Test Problem—Gaussian Fits for S Using Backward Difference

- Histogram of the relative sensitivity of the flux in $0.5623413 < E \leq 0.5956621$ MeV with respect to the block height for 250 pairs of realizations.



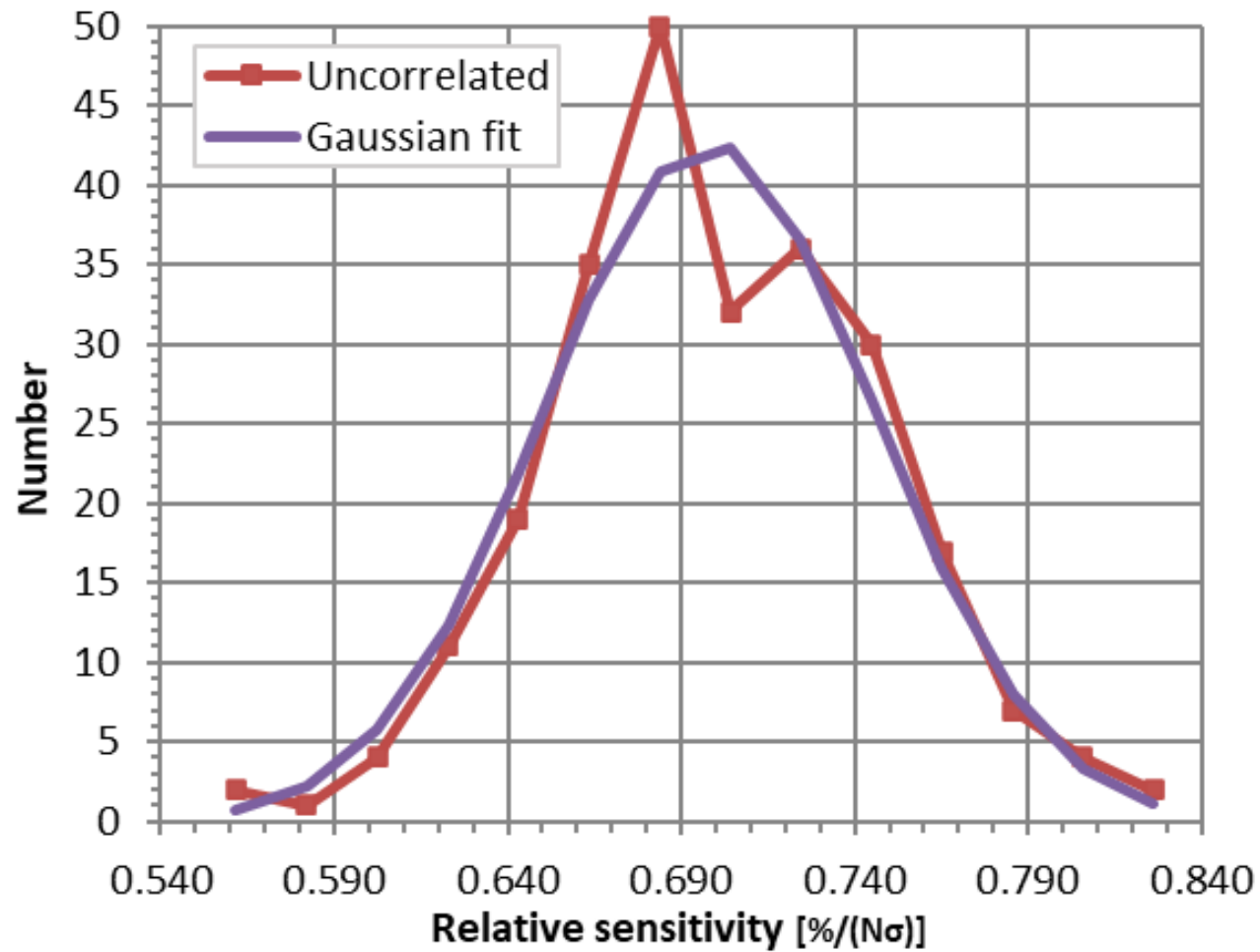
(a) Uncorrelated



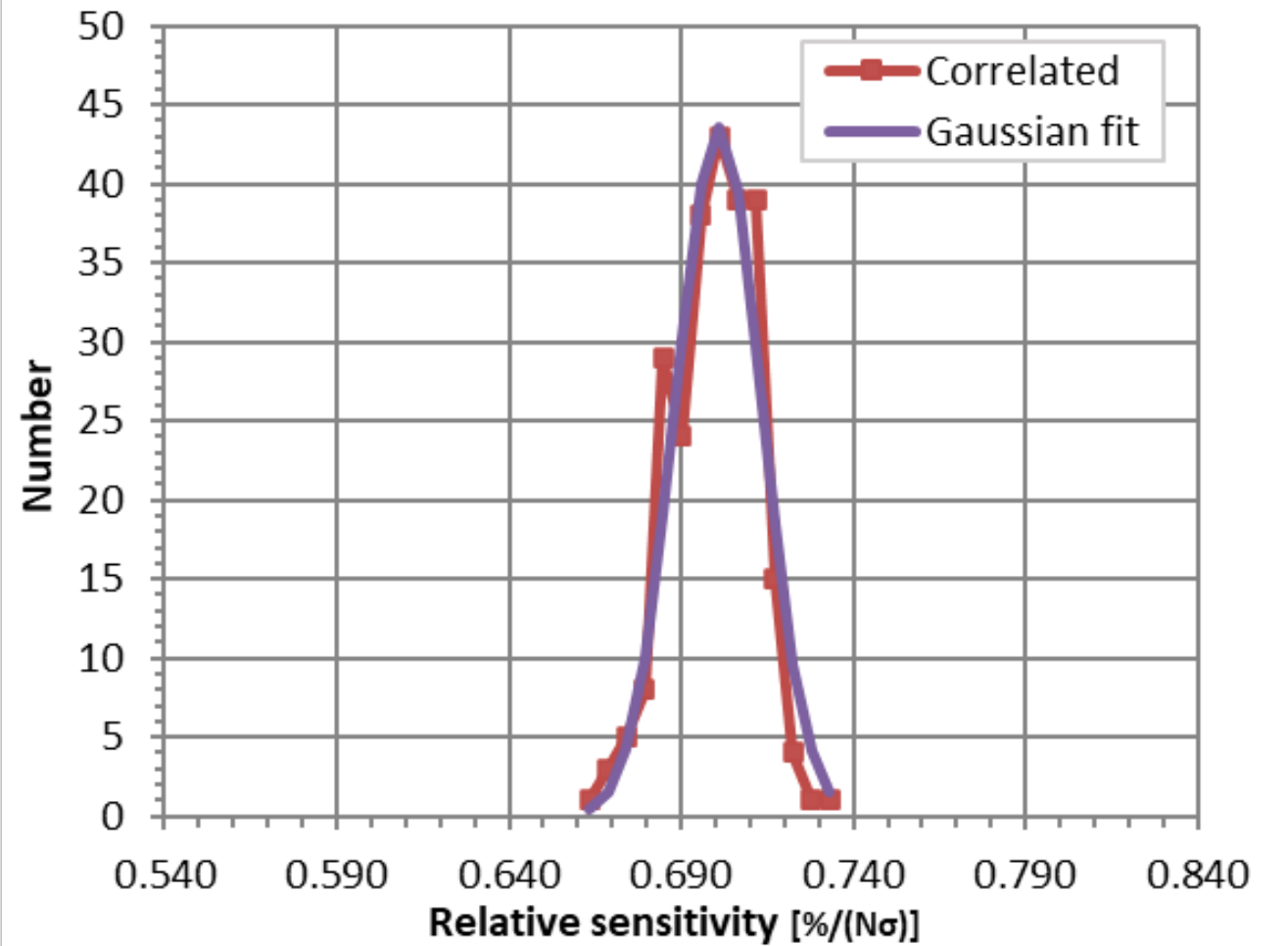
(b) Correlated

Koš'ál Test Problem—Gaussian Fits for S Using Forward Difference

- Histogram of the relative sensitivity of the flux in $0.5623413 < E \leq 0.5956621$ MeV with respect to the block height for 250 pairs of realizations.



(a) Uncorrelated



(b) Correlated

How Much Does All This Cost?

- The answer is always problem-dependent.
- Wall clock times for the Košťál test problem (2×10^9 histories, 244 processors, MPI only):

	20 TFC Entries (s)	100 TFC Entries (s)	100/20	1000 TFC Entries (s)	1000/20
Base Case	5113	5263	1.029	6210	1.215
Positive Perturbation	5327	5335	1.002	6347	1.191
Negative Perturbation	4957	4971	1.003	5972	1.205

Summary and Conclusions

- You can use correlated sampling with batch statistics from the TFC to reduce the uncertainty of many combinations of tallies.
 - + You *should* use this to compute the correct uncertainty for some combinations, such as a sum or ratio.
- You should use at least 100 TFC entries.
- Always use “DBCN 49j 2”.
- I have written a program that does batch statistics for you (it doesn't do sums).
 - + COSUBS = COrelated Sampling Using Batch Statistics
 - + My hope is to distribute this with MCNP Tools. It may be through github.
- Some of you can modify MCNP to increase the number of TFC entries.
- If not, you can still get 20 TFC entries per input file, and my code handles multiple output files.
 - + 5 runs will get you 100 TFC entries.
- I verified that

```
cosubs -b HeSu0_s00##_1e6.o -p HeSu1p_s00##_1e6.o -n HeSu1m_s00##_1e6.o -r .01 -o out1e6
```

with 50 output files gives the same results to 5 decimal places as

```
cosubs -b HeSu0_5e7.o -n HeSu1m_5e7.o -p HeSu1p_5e7.o -r .01 -o out5e7
```

Future Work

- COSUBS does not handle tally bins that are not in a TFC—e.g., a spectrum.
 - + COSUBS could be extended to read regular MCNP6 tally output.
 - + You can use the 2nd entry on PRDMP to print tallies 100 times during the run.
 - + It would be better if there were more digits in the regular tally output (easy to do in a modified code).
- COSUBS does not handle non-Boltzmann post-processed responses—e.g, Feynman Y or Sm_2 .
 - + You would need at least 100 input files.
 - + You do not need to propagate the uncertainties of the response for each realization.