

MCNP User Symposium 2021

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Advances in Monte Carlo Criticality Methods

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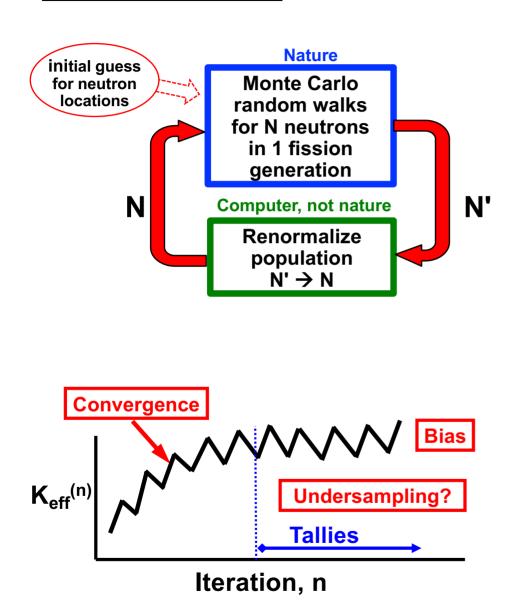
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Advances in MC Criticality Methods

- History & Concerns
- Meshing
- Fission-Matrix
- Acceleration
- Convergence Testing
- Population Size Testing
- Automation
- Examples
- Conclusions

History & Concerns

MC Criticality Calculations - Concerns



Bias in Keff

Nonconservative, \propto -1 / (neutrons/cycle)

Bias in source shape

Too low in high-importance regions, Too high in low-importance regions

Undersampling/clustering
 Not enough neutrons/cycle to cover space

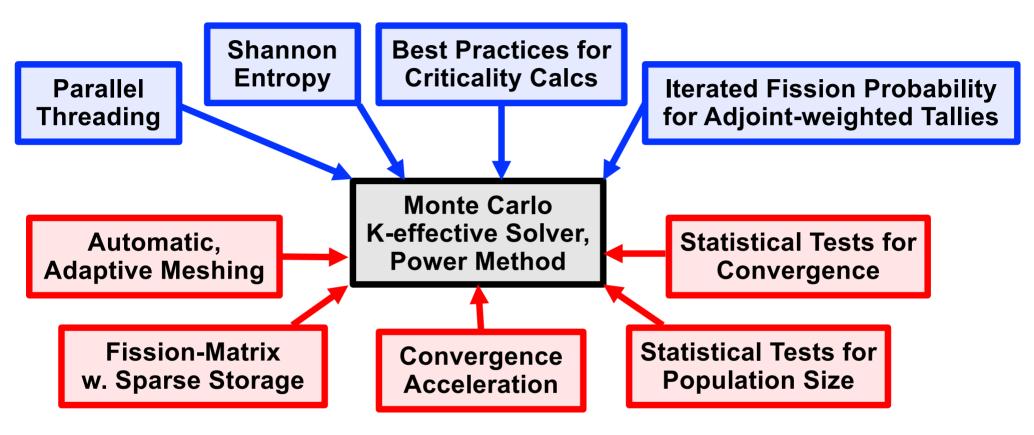
Convergence Source shape takes longer than keff

• Underestimate statistics Typically by 3-5x for local tallies

Best Practices

Source in all fissile regions. Examine H_{src} plot for convergence. >10k neuts/cycle (>100k big probs). A few 100 active cycles, or more

MC Criticality Calculations – History





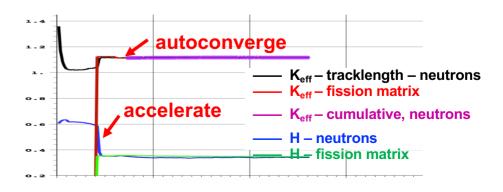
Automated Acceleration & Convergence Testing for MC Criticality

Enabling technology, automate & combine new methods

- Automated, adaptive meshing
 - Physics-based mesh used for Shannon entropy & fission-matrix
- Fission-matrix with adaptive sparse storage
 - Reference solution for global fission distribution
- Accelerate convergence of neutron distribution
 - Importance sampling of cycle sources, based on fission-matrix eigenfunction
- Statistical tests for convergence
 - 8 tests on metrics, 3 tests on distributions
 - Automatically begin active cycles & tallies
- Population size tests

Eliminate user burdens:

- Before: Trial run, eye-ball entropy plots, judgement, fix input, re-run
- Now: Fully automated, faster convergence, 11 convergence checks 2 population size checks
- Quantitative evidence of convergence
- Enables parameter studies & coupled multiphysics
- Saves significant computer time & people time
- Easy to use no user input needed



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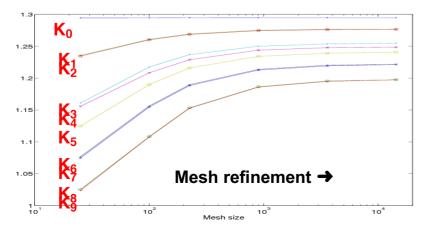
Meshing, **Fission-Matrix**, Acceleration, **Convergence Testing**, **Population Size Testing**, **Automation**

Meshing

• Since early-2000s, a mesh has been used for calculating Shannon entropy

(1)

- HSRC card or automatic meshing
- Auto-mesh was crude, ad hoc (but works)
- Today, mesh is used for:
 - Shannon entropy
 - Fission-matrix
 - Convergence acceleration & testing
 - Population size tests
- From MCD 2013, Sun Valley:
 - Detailed solutions for higher eigenmodes
 - Mesh was refined until higher-mode eigenvalue spectrum converged



- Today, for acceleration & convergence testing of global eigenfunction:
 - Only need global solution for fundamental mode
 - Physics-based metric to automatically choose mesh resolution:
 - L_{Fiss} = RMS distance from birth to next-generation fission
- Mesh resolution
 - For previous detailed analysis of higher-mode eigenfunctions,

 Δ {x,y,z} ~ .1 · L_{Fiss} works best

 For global convergence of fundamental eigenfunction,

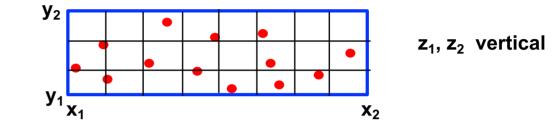
 Δ {x,y,z} ~ L_{Fiss} works best

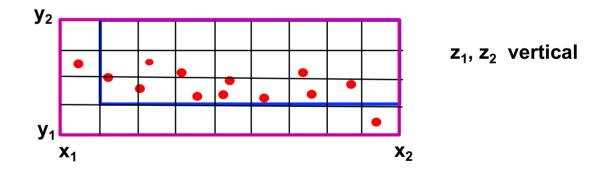
Meshing

Cycle 1 – set initial mesh

(2)

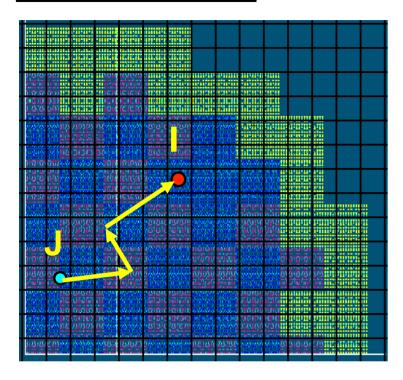
- Compute L_{Fiss}
- Bounding-box
- Set n_X, n_Y, n_Z for mesh spacing of ~ L_{fiss}
- Later cycles
 - Extend mesh if needed
 - Never shrink the mesh
 - Add same-size cells
 - Reallocate/reindex tallies for entropy, source, & fission-matrix





- Initial mesh & later extensions are automated no user input needed
- Mesh is used for tallying S_{neut}, H, F_{IJ}, & storing S_{FM}
- Meshing does not affect MC simulation or tallies

Fission-Matrix



- Mesh
 - Does not have to be aligned with material or geometric boundaries
 - OK for underlying geometry to be hexagonal, spherical, cylindrical, ...
 - Must cover all fissionable regions
- Example

 $100 \times 100 \times 100$ mesh = 10^6 regions F is $10^6 \times 10^6$, 8 TB for full matrix 10^{12} tallies for Green's function Adaptive, sparse-storage is necessary

- From neutron simulation in a cycle, tally $F(I \leftarrow J)$
 - Probability that neutron born in region J produces next-generation neutron in region I
 - Accumulate over all cycles (even inactive)
 - Estimate of point-to-point Green's functions
- For $n_x \cdot n_y \cdot n_z$ mesh
 - $N = n_x \cdot n_y \cdot n_z$ mesh cells
 - F-matrix is N N, large & nonsymmetric
 - Adaptive sparse-storage is used for F-matrix
- Fission-matrix equation for fission neutron source:

 $S = 1/k F \cdot S$

Given F, can solve for k & S.

Provides reference solution for acceleration & convergence testing

Accelerating Source Convergence

At the end of each cycle

- S_{FM} is available fission-matrix eigenfunction at end-of-block
- S_{neuts} is available actual neutron source at end-of-cycle
- During inactive cycles, can optionally use (S_{FM} / S_{neuts}) for importance sampling of the fission source, for each mesh region
 - Pushes neutron distribution toward F-matrix reference
 - Recomputed each cycle using S_{FM} from previous end-of-block, and S_{neuts} for current end-of-cycle
 - Works typically reduces inactive cycles by 2-20 X

- Further development under consideration:
 - Investigate using $S_{FM}^{adjoint}$ for source importance sampling
 - Maybe coarsen the fission-matrix, to reduce statistical noise

Statistical tests for convergence

Slope test

For a block of cycles (default = 10)
Not Converged

For result x from each cycle in block, compute least-squares slope & σ_{slope}

|slope(x)| < 0.0001 → pass, slope ~ 0
|slope(x)| < t_{0.025} σ_{slope} → pass, slope ~ 0
within statistics

Metric tests, at end-of-block for

convergence testing

- 1. Slope K_{tracklen}
- 2. Slope K_{collide}
- 3. Slope K_{absorb}
- 4. Slope H, Shannon entropy
- 5. Slope H_x, entropy X marginal
- 6. Slope H_Y, entropy Y marginal
- 7. Slope H_z, entropy Z marginal
- 8. H_{block} within 1% of H_{FM}

If Test 8 passes, strong evidence of convergence If Test 8 fails, ignore it – might be low popsize

- Distribution tests, at end-of-block for convergence testing
 - 9. Kolmogorov-Smirnov test at 95% level, S_{block} & S_{FM} have same distrib.

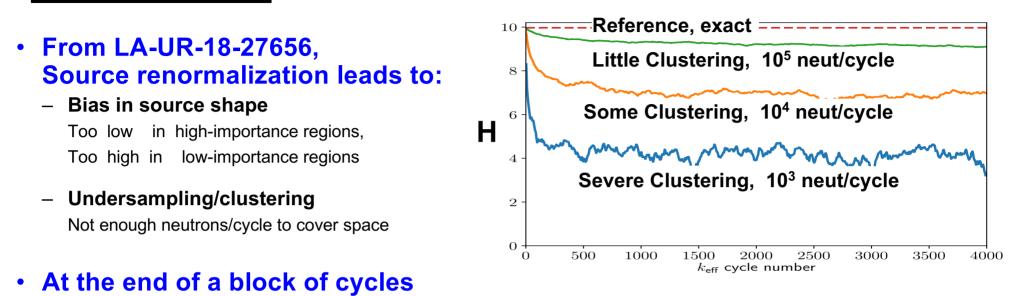
For multi-D distributions, KS statistic depends on ordering. Take worst case KS statistic for many random permutations.

- 10. Chi-square 2-point test at 95% level, S_{block} & S_{FM} have same distrib.
- 11. Relative entropy (Kullback-Liebler discrepancy) test at 95% level for S_{block} & S_{FM}

If Test 11 passes, strong evidence of convergence If Test 11 fails, ignore it – might be low popsize

- If convergence tests all pass, convergence is locked-in
 - Tests continue for each block
 - Some tests may later fail (due to statistics), but convergence not rescinded
 - Start the tallies

Statistical tests for Population Size



- S_{FM} is available source from fission-matrix fundamental at end-of-block
- S_{block} is available neutron source accumulated in mesh during block
- S_{FM} can be considered a reference solution
- If population size is large enough such that source renormalization bias is negligible,

S _{block} ~ S _{FM} ,	compare distributions using relative entropy
< H(S _{cycle}) > _{block} ~ H(S _{FM}),	compare FM entropy to neutron entropy averaged over cycles in block

Automated Methods

- Cycle 1
 - Estimate L_{Fiss} & set initial mesh
- Initial cycles
 - Iterate until mesh and S_{neut} & F tallies are stable
 - Automated, test that (Δ nonzero tallies) < 2%, 5%
 - When S_{neut} & F tallies are stable, begin a block of cycles
- At each end-of-block of cycles (default = 10 cycles/block)
 - Solve F-matrix equations for S_{FM} , fundamental mode eigenfunction
 - Convergence tests
 - 11 statistical tests must all pass for convergence
 - If converged, set active cycles to begin with next cycle, start population size tests after next block
 - If <u>not</u> converged, <u>accelerate source convergence</u> for each cycle by importance sampling with weights:

 $S_{FM}(m) / S_{neut}(m), m = bin$



Extra MCNP input to activate new features:

kopts fmat= yes fmatconvrg= yes fmataccel= yes

kcode

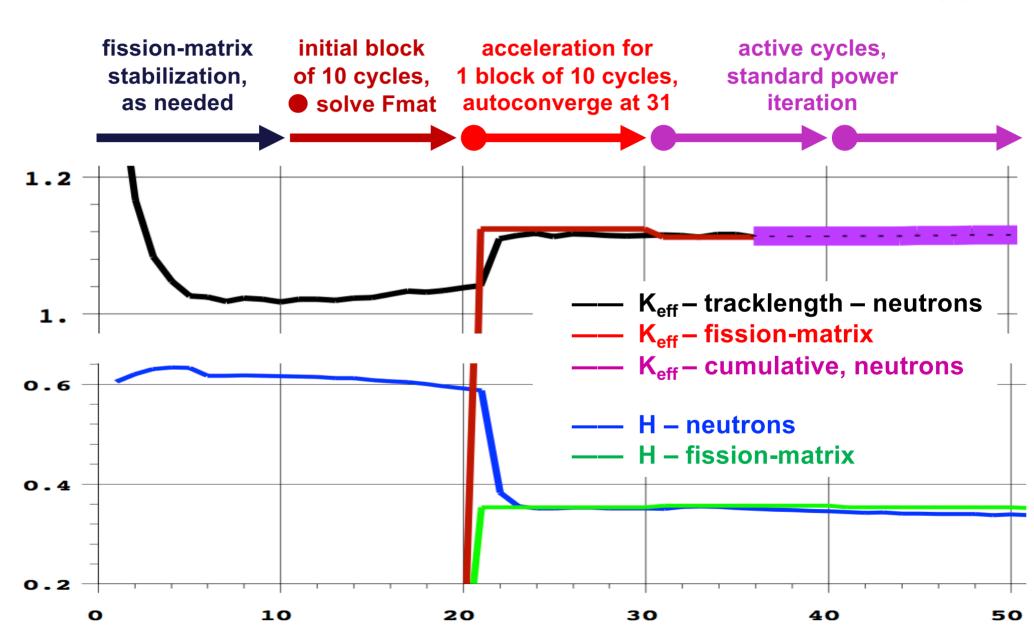
- Must use 10,000 or more neutrons/cycle
- 3rd entry ignored, number of inactive cycles
- Active cycles after convergence (kcode_{4th entry} kcode_{3rd entry})

Other Notes

- Another 1 or 2 input kopts entries are needed for
 - 1D or 2D problems (with infinite extent in 1 or 2 dimensions)
 - Precise Dominance Ratio calculations

Example: OECD-NEA Source Convergence Problem TEST4S

50,000 neuts/cycle, acceleration, auto-converge, k = 1.1165 (2)



```
comment. --
comment. The MESH (adaptive, axis-aligned, cartesian) to be used for computing comment. Shannon entropy, fission-matrix tallies (if used), and source comment. convergence checking is initially defined by:
comment. max mesh spacing for automesh = 1.0052E+01
                                                                                                                                                                                                                    Fiss
comment.
                                  total mesh cells = 3675
comment.
comment.
                                                                                                                                                                                                                 Boundina
                                  Xbins=
Ybins=
                                                         35
35
                                                                      Xmin=-1.6861E+02
Ymin=-1.6856E+02
                                                                                                                       Xmax= 1.6856E+02
Ymax= 1.6857E+02
                                                                                                                                                                         dx= 9.6334E+00
dy= 9.6323E+00
comment.
comment.
                                                                                                                                                                                                                   Box
                                                            3
                                                                                                                                                                         d\bar{z} = 6.5344E+00
                                  Zbins=
                                                                      Zmin = -9.6460E + 00
                                                                                                                        Zmax = 9.9571E + 00
comment.
comment.
                                                                                                                                                                                                               Initial Mesh
comment. the mesh will be automatically extended if necessary,
comment. preserving the original mesh cells and spacing.
comment.
comment. --
                       FISSION MATRIX WILL BE COMPUTED to estimate dominance ratio,
comment.
                                     based on fission sites only - not flights or collisions
comment.
comment.
                          The mesh for the fission matrix is the same as the entropy mesh, using 3675 mesh bins for tallying fission neutrons
comment.
comment.
comment.
                                    Fission matrix mesh will be extended if
comment.
                                             any fission sites are found outside this mesh.
comment.
comment.
                                     Fission matrix tallies will be reset after cycle
comment.
                                    Fission matrix eigenfunction will be found every 10 cycles.
comment.
comment.
                                    Fission matrix dimensions: 3675 x 3675
comment.
comment.
                                    Compressed-row-storage is used for the fission matrix.
max number of nonzero entries: 13505625
comment.
comment.
comment.
comment.
                          FMATCONVRG option is being used.
Statistical tests on the neutron & fiss-matrix distributions
comment.
comment.
                                  will be used to determine convergence & begin active cycles.
comment.
                                  The 3rd entry on the KCODE card may be ignored.
comment.
comment.
                                  Targets for statistical tests:
comment.
                                       h slope: < 0.95 conf level, or < 0.0001
k-slope: < 0.95 conf level, or < 0.0001
distribs: < 0.95 conf level, h_diff: < 0.01
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```

cycle 1	k(col) 1.35733	ctm 0.04	entropy 0.60521	active k(col) std dev	chains 35416	
2	1.16857	0.10	0.62080	extend H-mesh		22433	
3	1.08223	0.13	0.63109	extend H-mesh dS= 3%, dF= 3	to: 37 x 35 x 4 34%, shift window	17100	
4	1.05100	0.17	0.63410	extend H-mesh	•	13800	
5	1.02827	0.21	0.63348	extend H-mesh		11529	Source, fission-matrix,
6	1.02118	0.25	0.61732		to: 37 x 37 x 5 10%, shift window	9997	& mesh
7	1.02018	0.29	0.61762		9%, shift window	8746	stabilization
8	1.02413	0.32	0.61845	·	7%, shift window	7790	
9	1.01974	0.37	0.61766	dS= 0%, dF=	7%, shift window	6974	
10 11	1.01709 1.02129	0.43 0.48	0.61656 0.61606	dS= 1%, dF=	5%, shift window	6313 5815	
12	1.01705	0.53	0.61452	dS= 1%, dF=	5%, shift window	5351	
13 14	1.02459 1.02193	0.58 0.65	0.61263 0.61214			4975 4640	Block
15 16 17	1.02741 1.03005 1.03266	0.70 0.73 0.78	0.60894 0.60600 0.60435			4372 4091 3852	of
18 19	1.03369 1.03485	0.83 0.87	0.60065 0.59622			3628 3426	cycles
20 21	1.03631 1.04159	0.91 0.96	0.59177 0.58774			3245 3074	

fmatrix keff= 1.12401, DR= 0.91098, iters=

199 🗲 from F-matrix solution

fmatrix keff= 1.12400, DR= 0.91098, iters= 199 **CONVERGENCE INFO & CHECKS:** (based on last 10 cycles) entropy for fmatrix eigenvector = 0.35378dif= 66.13% entropy for neutron last cycle = 0.58774relative entropy for last cycle = 2.06900 slope of keff (tracklen) = 2.0E-03, target: < 5.3E-04FAIL slope of keff (collide) target: < 5.3E-04FAIL = 2.1E-03,slope of keff (absorb) = 2.0E-03, target: < 5.8E-04FAIL = -2.6E-03, target: < 4.3E-04slope of entropy FAIL target: < 5.1E-04slope of entropy X marginal = -2.1E-03, FAIL slope of entropy Y marginal = -2.1E-03, target: < 4.2E-04FAIL slope of entropy Z marginal = 8.7E-04, target: < 3.3E-04FAIL entropy dif, neuts vs fmat = 7.1E-01, target: < 1.0E-02n/a Kolmo-Smirnov, distrib, stat = 6.8E-01, target: < 9.1E-02 FAIL Chi-square, distrib, stat = 5.0E+04, target: < 5.1E+02 FAIL rel-h-block, distrib, stat = 2.5E+00, target: < 5.1E-03n/a

***** convergence tests were NOT passed *****

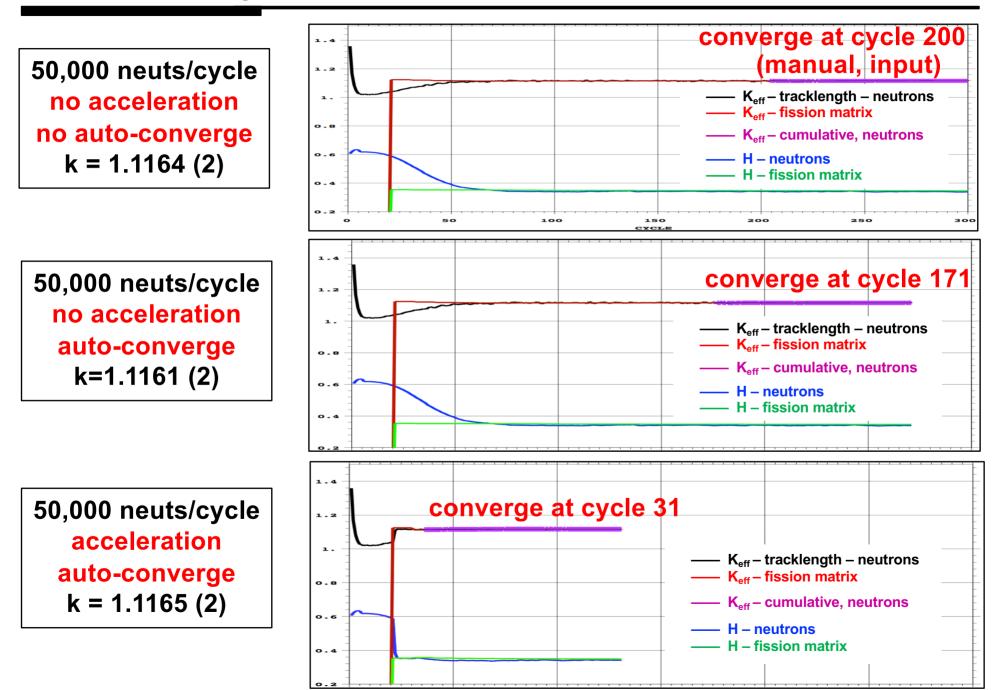
MISCELLANEOUS INFO & CHECKS:

rmse	=	1.16	8	
fmat nnz=		11884,	0.09	%

22	1.10782	0.81	0.38309
23	1.11376	0.85	0.35605
24	1.11583	0.88	0.35129
25	1.11726	0.92	0.35104

accelerate:	Imin=	0.2,	Imax=	4.7	
					2134
accelerate:	Imin=	0.2,	Imax=	3.8	
					1499
accelerate:	Imin=	0.2,	Imax=	3.2	
					1233
accelerate:	Imin=	0.2,	Imax=	5.0	1055
		• •	_	• •	1077
accelerate:	Imın=	0.2,	Imax=	3.4	

31 1.11257 1.12 0.35069 680 138 fmatrix keff= 1.11187, DR= 0.91653, iters= **Dominance Ratio** estimate (based on last 10 cycles) **CONVERGENCE INFO & CHECKS:** entropy for fmatrix eigenvector 0.35656 = entropy for neutron last cycle 0.35069 dif= -1.65% = relative entropy for last cycle 0.00972 = slope of keff (tracklen) 4.2E-03, target: < 5.1E-03PASS = < 4.9E-03slope of keff (collide) = 4.6E-03, target: PASS Quantitative slope of keff (absorb) 4.6E-03, target: < 4.9E-03PASS = slope of entropy = -1.4E-02, target: < 1.6E-02PASS Evidence target: < 1.9E-02slope of entropy X marginal = -1.8E - 02, PASS For slope of entropy Y marginal target: < 1.9E-02= -1.8E-02, PASS = 1.3E-03, target: < 1.6E-03slope of entropy Z marginal PASS Convergence target: < 1.0E-02entropy dif, neuts vs fmat = -9.1E - 04, PASS Kolmo-Smirnov, distrib, stat = 2.5E-03, target: < 9.1E-02PASS Chi-square, target: < 5.1E+02 distrib, stat = 9.0E+01, PASS < 5.1E-03PASS rel-h-block, distrib, stat = 2.8E-03, target: cycles * * FISSION SOURCE HAS CONVERGED, based on last 10 * * * * * * Metrics: * * slope of keff (tracklen) * * (within uncert) is O * * slope of keff (collide) is O * * (within uncert) Quantitative * * slope of keff (absorb) is O * * (within uncert) * * slope of entropy is O (within uncert) * * Evidence * * slope of entropy X marginal is O * * (within uncert) For * * slope of entropy Y marginal is O * * (within uncert) ** * * slope of entropy Z marginal is O (within uncert) Convergence ** entropy dif, neuts vs fmat is O (within uncert) * * * * * * **Distribution** checks: * * Kolmo-Smirnov, distrib, stat, neut vs fmat (within conf) * * ** distrib, stat, neut vs fmat (within conf) * * Chi-square, * * neut vs fmat (within conf) * * distrib, stat, rel-h-block, Convergence is locked-in, even if some tests fail in future cycles Active cycles will begin with cycle = 32 Active cycles will end with cycle = 131 Total active cycles to be run 100 =



Summary & Conclusions

Summary & Conclusions

- Automation, user convenience, & correctness are major requirements
- Statistical testing for iteration convergence
 - No <u>single</u> statistical test is adequate, use many tests for robustness
 - Slope test is a powerful tool, can be applied to any metric vs cycle
- Statistical testing for population size
 - Very new, much potential for additional testing approaches
- Practical considerations
 - These methods work on a wide variety of application problems
 - Correct results for every problem tested, never converged to a wrong answer
 - Are the methods guaranteed to work correctly?
 - Assumes a reasonable initial source guess, covering fissionable regions
 - Of course, contrived & devious problems can be constructed to break things
 - Fall-back is the conventional 1950s-style methods, which have not changed
 - Significant advance in state-of-the-art for MC criticality
 - Prevent user-errors related to iteration convergence
 - Quantitative convergence evidence for reviews & audits
 - Auto-convergence for routine work, parameter studies, multiphysics, ...

References

- F.B. Brown (Ed.), (11 coauthors US, UK, FR), "Statistical Tests for Diagnosing Fission Source Convergence and Undersampling in Monte Carlo Criticality Calculations", final report from OECD-NEA-WPNCS Subgroup-6 (to be published by OECD-NEA in 2021)
- F.B. Brown, S.E. Carney, B.C. Kiedrowski, W.R. Martin, "Fission Matrix Capability for MCNP, Part I Theory", Mathematics & Computation 2013, Sun Valley, ID, LA-UR-13-20429 (2013).
- S.E. Carney, F.B. Brown, B.C. Kiedrowski, W.R. Martin, "Fission Matrix Capability for MCNP, Part II Applications", Mathematics & Computation 2013, Sun Valley, ID, LA-UR-13-20454 (2013).
- F.B. Brown, C.J. Josey, S. Henderson, W.R. Martin, "Automated Acceleration and Convergence Testing for Monte Carlo Criticality Calculations", ANS M&C 2019, Portland OR, LA-UR-19-20308 (2019)
- F.B. Brown, C.J. Josey, S. Henderson, W.R. Martin, "Automated Acceleration and Convergence Testing for Monte Carlo Nuclear Criticality Safety Calculations", ICNC 2019, Paris FR, LANL report LA-UR-19-20482 (2019)
- F.B. Brown, "A Review of Best Practices for Monte Carlo Criticality Calculations", ANS NCSD 2009, Hanford WA, LA-UR-09-03136 (2009).
- F.B. Brown, "Investigation of Clustering in MCNP6 Monte Carlo Criticality Calculations", Int. Conf. on Transport Theory, Monterey CA, Oct 2017, LA-UR-17-29261 (2017).
- F.B. Brown, W.R. Martin, "Statistical Tests for Convergence in Monte Carlo Criticality Calculations", LA-UR-18-28764 (2018).
- F.B. Brown, C.J. Josey, "Diagnostics for Undersampling and Clustering in Monte Carlo Criticality Calculations", LA-UR-18-27656 (2018).
- F.B. Brown, "Monte Carlo Techniques for Nuclear Systems", LA-UR-16-29043 (2016).
- F.B. Brown, "Advanced Computational Methods for Monte Carlo Calculations", LA-UR-18-20247 (2018)

Additional Examples

(as time permits)

MCNP6.3 Test Problems for Fission Matrix Based Automated Convergence & Acceleration of K-eigenvalue Problems

- VALIDATION_CRITICALITY benchmark suite
- Godiva bare HEU sphere
- PWR2d commercial PWR
- ATR INL advanced test reactor
- AGN-201m UNM research reactor
- C5G7 3D U-Mox benchmark, OECD-NEA
- Triga reactor
- ACRR Sandia burst reactor, with FREC
- LCT-078-001 Sandia critical experiment
- 3D PWR Hoogenboom-Martin benchmark, OECD-NEA
- Whitesides problem K-effective of the world model
- TEST4S simplified Whitesides, OECD-NEA
- FPOOL OECD-NEA source convergence benchmark 1

VALIDATION_CRITICALITY benchmark suite

- Standard MCNP validation suite since 2002 (Mosteller)
 - 31 ICSBEP Handbook problems, critical experiments
 - Run using ENDF/B-VII.1 nuclear data
 - Timing results include all I/O, input & xsec file processing, Monte Carlo random walks, printing results, etc. for all 31 problems

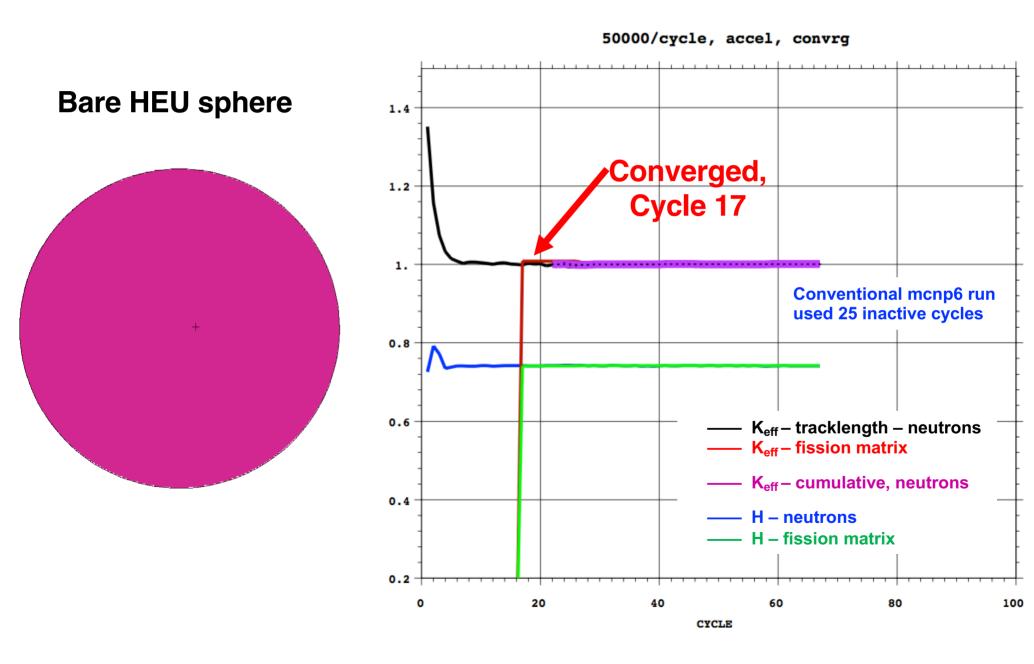
Timing tests

- 50,000 neutrons/cycle for all runs
- For standard runs, 100 inactive cycles, 100 active cycles
- For auto accelerate & converge,
 100 active cycles

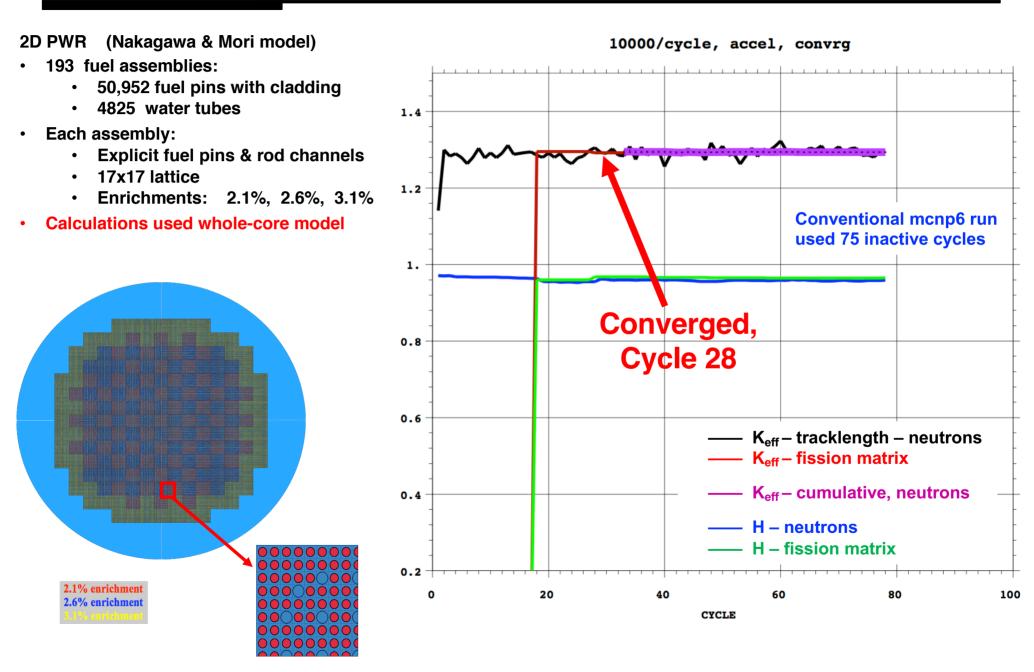
Standard run: 106 minutes

Auto accel & converge: 70 minutes

Godiva Problem



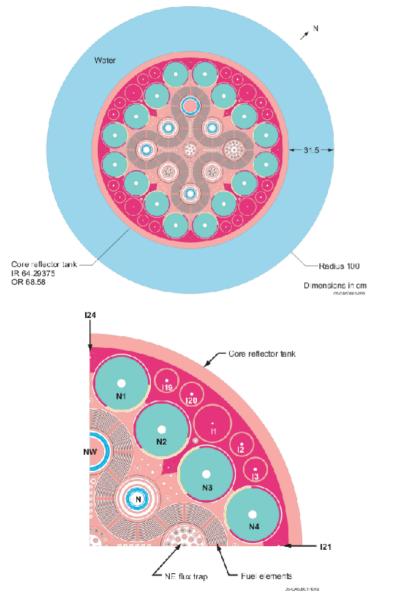
Whole-core 2D PWR Model

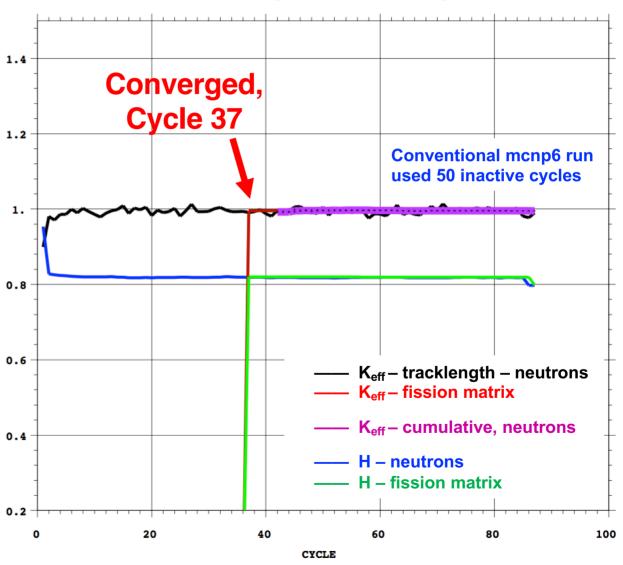


Advanced Test Reactor

"Serpentine Arrangement of Highly Enrichment Water-Moderated Uranium-Aluminide Fuel Plates Reflected by Beryllium"

25000/cycle, accel, convrg



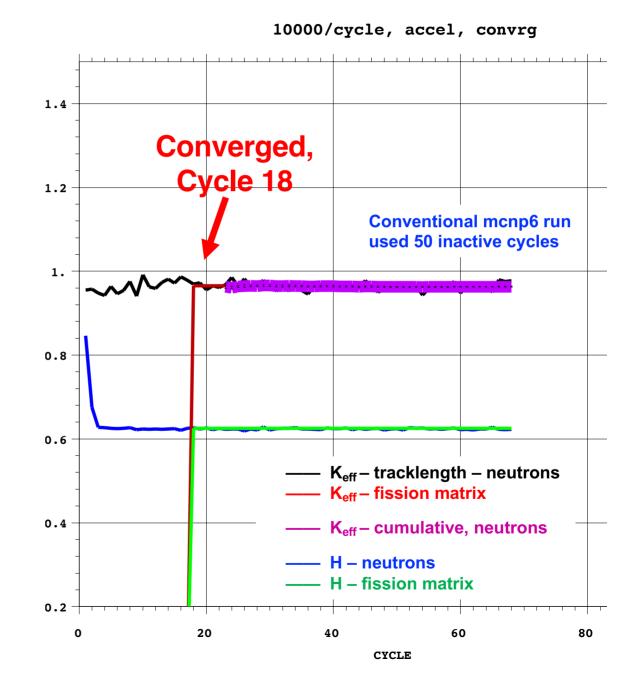


AGN-201 - Univ. New Mexico Research Reactor

UNM Research Reactor

AGN

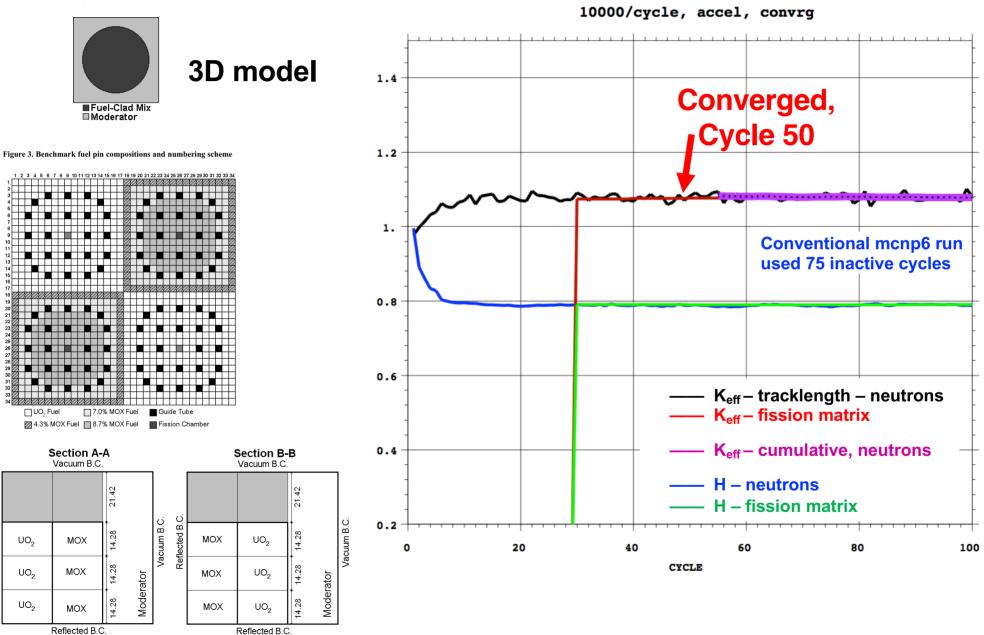




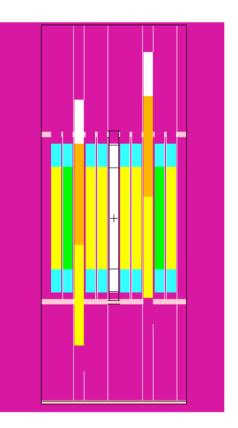
OECD-NEA Benchmark - C5G7

Figure 2. Fuel pin layout

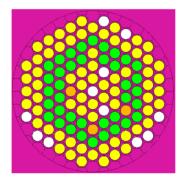
Reflected B.C.

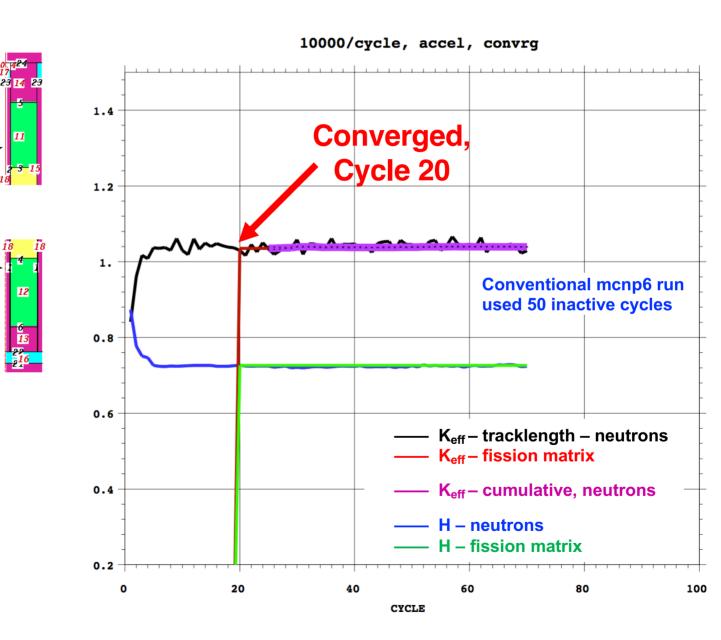


TRIGA Reactor

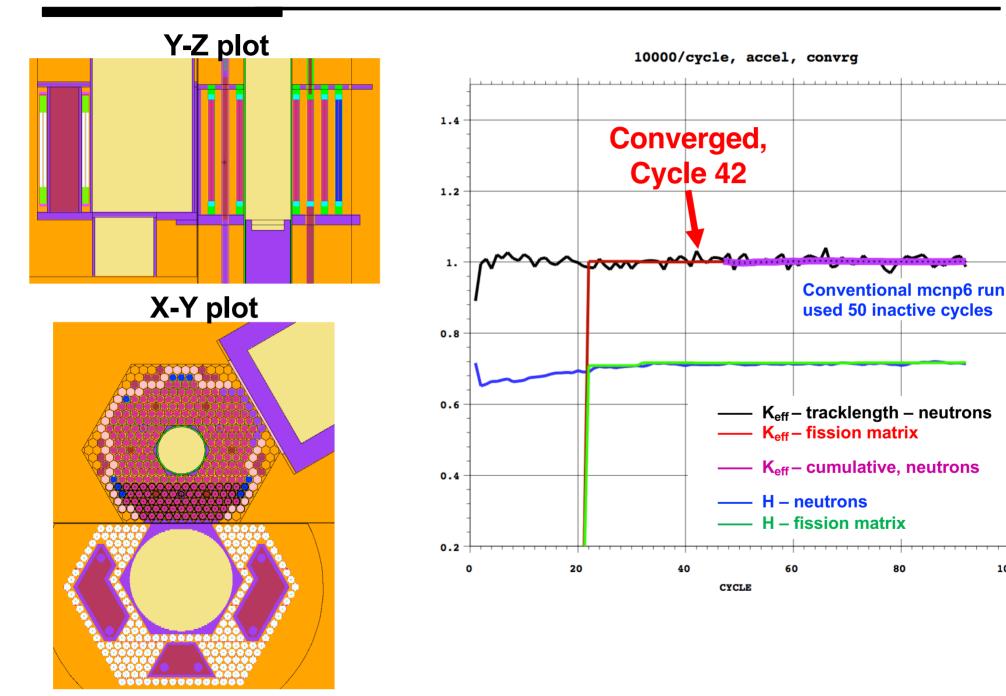


18



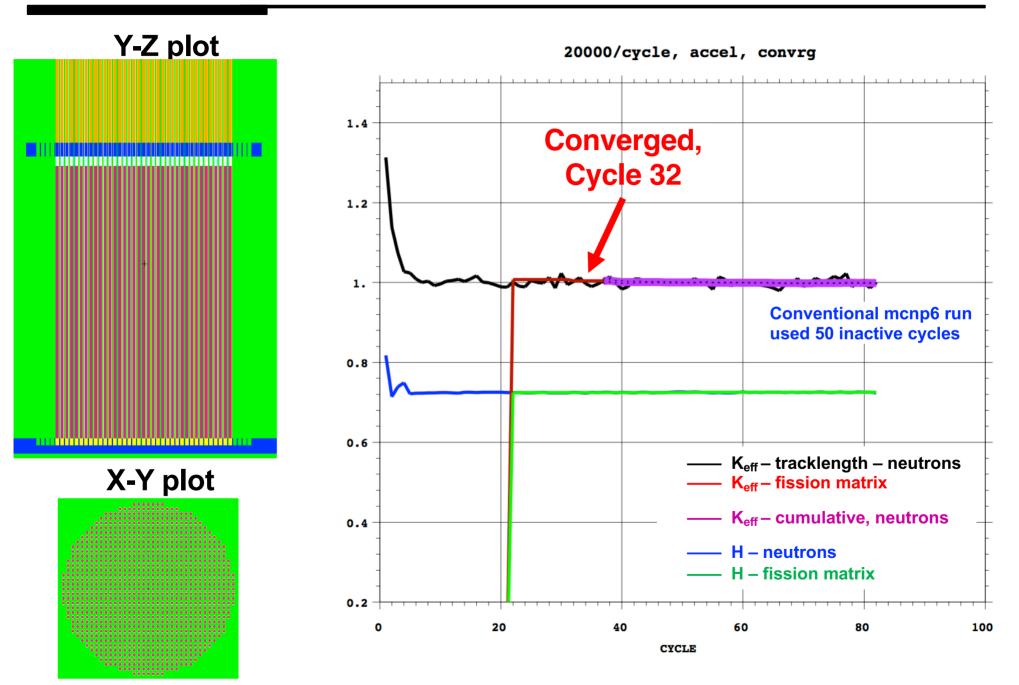


Sandia burst reactor - ACRR, with FREC

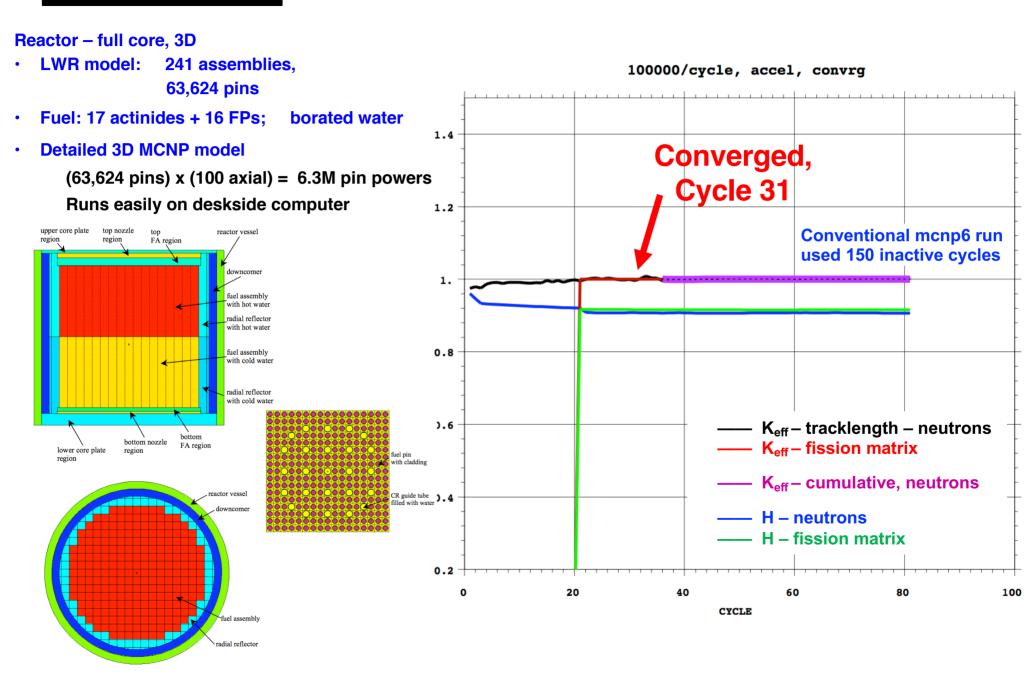


100

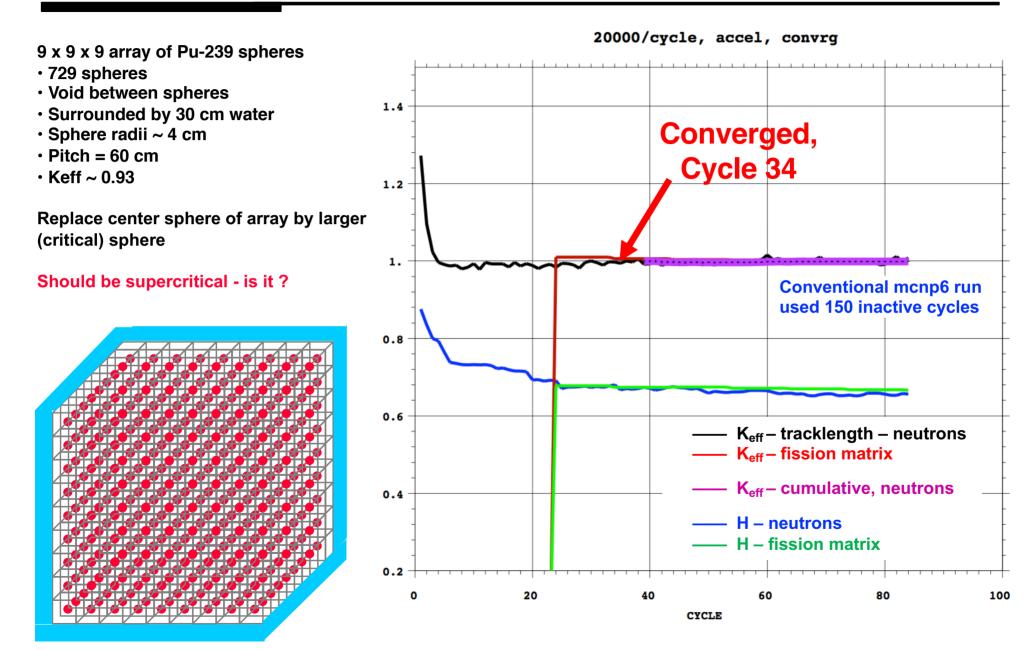
Sandia critical experiment – LCT-078-001, 1,057 rod assembly



OECD-NEA "Hoogenboom-Martin Performance Benchmark"



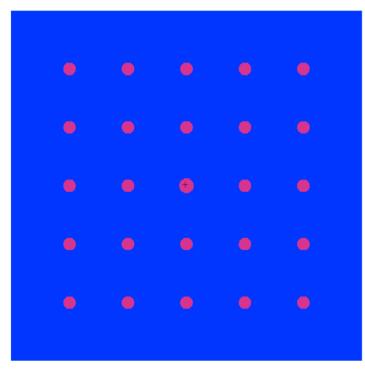
Whitesides' Model Problem – K-eff of the World

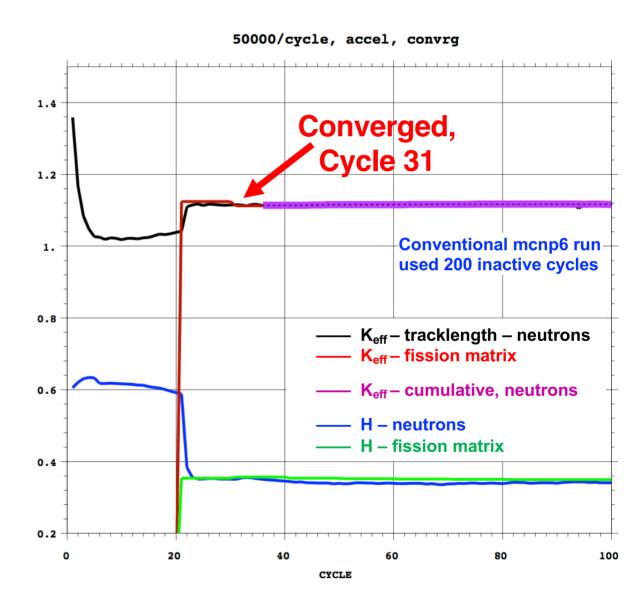


OECD-NEA Source Convergence Problem TEST4S

OECD-NEA source convergence benchmark

- Simplified version of Whitesides problem
- 5 x 5 array of HEU spheres
 - center sphere, R = 10 cm
 - others, R = 8.71 cm
 - pitch = 80 cm
 - air in between spheres
 - vacuum boundary conditions





OECD-NEA Fuel Storage Pool

