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Predicting Monte Carlo Tally Variance and Calculation Time when Using Forced-flight Variance Reduction—Theory

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Outline

Introduction & Terminology

Objective: perform computational-cost optimization of Monte Carlo calculations using forced-flight variance reduction

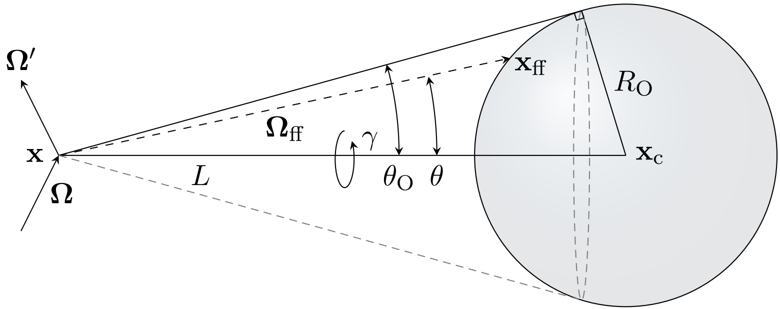
- Process overview
  - Deduce biasing operator(s) for forced-flight variance reduction
  - Construct history-score probability density function (HSPDF, $\psi\left(\mathbf{p},s\right)$)
  - Derive history-score moment equations (HSMEs, $\Psi_{1,2}\left(\mathbf{p}\right)$)
  - Construct future-time probability density function (FTPDF, $\upsilon\left(\mathbf{p}\right)$)
  - Derive future-time equation (FTE, $\Upsilon\left(\mathbf{p}\right)$)
  - Calculate computational cost

$$\$\left(\mathbf{p}\right)=\frac{\left[\Psi_2\left(\mathbf{p}\right)-\Psi_1^2\left(\mathbf{p}\right)\right]\Upsilon\left(\mathbf{p}\right)}{\Psi_1^2\left(\mathbf{p}\right)}. \quad (1)$$

Forced-flight Variance Reduction Overview

- Forced-flight: directly biases particle direction of flight
- "DXTRAN" in the MCNP code
  - Developed in the late 1970s, not clear by who
  - Best reference: MCNP theory manual (X-5 Monte Carlo Team, 2008)
- "Forced flight" in the MCBEND code
  - (Chucas et al., 1994; Shuttleworth et al., 2000)
- "Forced scattering" in a research version of the EGSnrc code
  - (Tickner, 2009; Kawrakow et al., 2017)
- Diverse fields of use
  - Medical physics
  - Research reactor irradiation beam port flux characterization
  - Neutron generator device design
  - Spacecraft nuclear propulsion
  - Inertial confinement fusion reactor design

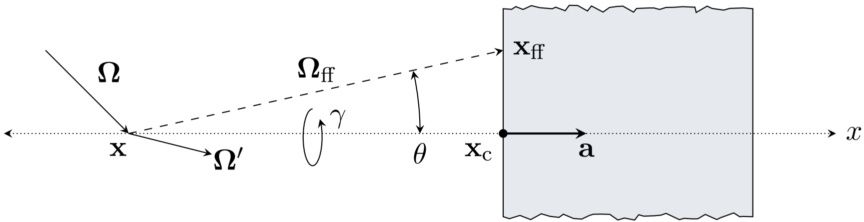
Simplified Spherical Forced-flight Variance Reduction



- User-defined geometry cell-wise rouletting fraction, β_c (i.e., `DXC`)
- Occurs following every valid non-absorptive collision
  - Weight balance by terminating particles entering the sphere
- No rouletting on transmission for weight control; no weight cutoffs

$$w_{\mathrm{ff}} = \frac{w}{\beta_c} \frac{g(\mu)}{\tilde{g}(\mu)} \frac{\exp\left[-\lambda\left(\mathbf{x}, \mathbf{x}_{\mathrm{ff}}, E\right)\right]}{1} \tag{2}$$

1-D Cartesian Forced-flight Region

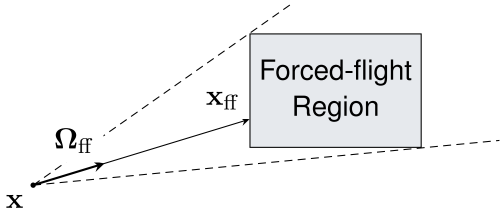


- User-defined geometry cell-wise rouletting fraction, β_c (i.e., `DXC`)
- Occurs following every valid non-absorptive collision
- No rouletting on transmission for weight control; no weight cutoffs

$$w_{\mathrm{ff}} = \frac{w}{\beta_c} \frac{g(\mu)}{\tilde{g}(\mu)} \frac{\exp\left[-\lambda\left(\mathbf{x}, \mathbf{x}_{\mathrm{ff}}, E\right)\right]}{1} \tag{3}$$

- Kulesza et al. (2018)

2-D Cartesian Forced-flight Region



- User-defined geometry cell-wise rouletting fraction, β_c (i.e., `DXC`)
- Occurs following every valid non-absorptive collision
- No rouletting on transmission for weight control; no weight cutoffs

$$w_{\text{ff}} = \frac{w}{\beta_c} \frac{g(\mu)}{\tilde{g}(\mu)} \frac{\exp\left[-\lambda\left(\mathbf{x}, \mathbf{x}_{\text{ff}}, E\right)\right]}{1} \tag{4}$$

- Kulesza et al. (2018)

HSME Methodology Overview

- History-score moment equations (HSMEs)
  - Describe Monte Carlo random walk statistical moments
  - HSME-based methods are Monte Carlo code agnostic
- Specific implementations are **directly** related to a Monte Carlo code
  - Descriptions of geometry, materials, tallies, sources, etc.
  - VR technique availability & implementation
  - Must deterministically model Monte Carlo transport code processes
- Different random walks available
  - Terminates with absorptive collision, continues with collision with emergence, continues with surface crossing
  - Modified as a function of variance reduction
- First, assemble history-score probability distribution function (HSPDF)
  - Probability of contributing score $\mathrm{d}s$ about s from $\mathbf{p}$

$$\psi\left(\mathbf{p}, s\right)\mathrm{d}s = \sum_i \psi_i\left(\mathbf{p}, s\right)\mathrm{d}s \tag{5}$$

HSME Components: Transport Operators

- HSPDF is constructed from **transport operators** and **scoring functions**
- Transport operators
  - Describe particles undergoing phase-space change

$\mathcal{T}(\mathbf{p}, \mathbf{p}')$	for particles undergoing free flight from $\mathbf{p}$ to $\mathbf{p}'$
$\mathcal{S}(\mathbf{p}, \mathbf{p}')$	for particles crossing a Monte Carlo surface (combining $\mathcal{S}$ and $\mathcal{A}$ from Kiedrowski et al. (2019))
$\mathcal{K}(\mathbf{p}, \mathbf{p}')$	for particles undergoing collision
$\mathcal{E}(\mathbf{p}, \mathbf{p}')$	for particles undergoing emergence (e.g., scattering) from a non-absorptive collision
$\mathcal{A}(\mathbf{p})$	for particles undergoing absorption, which is considered terminal in this work
$\mathcal{B}_{\text{ff}}(\mathbf{p}, \mathbf{p}')$	for the forced-flight process
$\mathcal{T}_{\text{ff}}(\mathbf{p}, \mathbf{p}')$	for free flight, with truncation, following forced-flight

HSME Components: Scoring Functions

- Scoring functions
  - Probability density functions giving history-scoring behavior
- Surface-crossing tally scoring function:

$$f_{\mathcal{S}}\left(\mathbf{p},s\right)=\delta\left(s-w\right)\,,\ \mathbf{x}\in\partial\Gamma_{\mathcal{S}}\,,\ \mathbf{\Omega}\cdot\mathbf{n}>0, \tag{6}$$

- Expected track-length tally scoring function (Solomon, 2010):

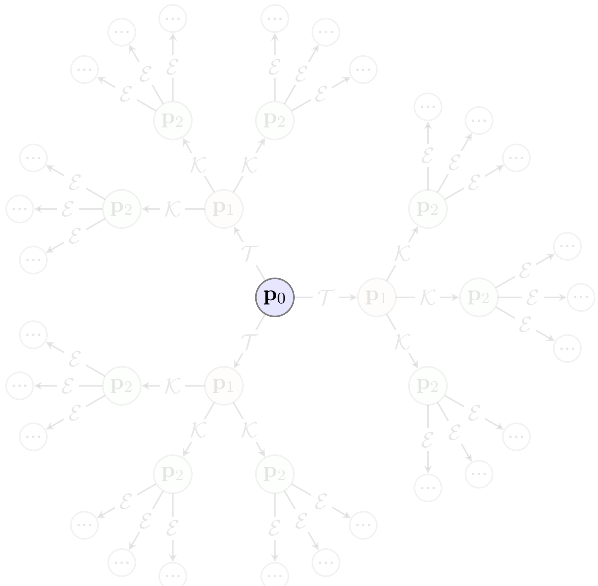
$$f_{\mathcal{T}}\left(\mathbf{p},s\right)=\delta\left(s-\frac{w}{V\Sigma_{\mathrm{t}}\left(\mathbf{x}\right)}\left[1-\exp\left(-d\left(\mathbf{x},\mathbf{\Omega}\right)\cdot\Sigma_{\mathrm{t}}\left(\mathbf{x}\right)\right)\right]\right)\,,\ \mathbf{x}\in\Gamma_{\mathcal{T}}, \tag{7}$$

- Construct HSPDF from transport operators and scoring functions, e.g., analog collision with emergence:

$$\psi_{\mathcal{E}}\left(\mathbf{p}_0,s\right)=\mathcal{T}\left(\mathbf{p}_0,\mathbf{p}_1\right)\mathcal{K}\left(\mathbf{p}_1,\mathbf{p}_2\right)\mathcal{E}\left(\mathbf{p}_2,\mathbf{p}_3\right)\times\int \mathrm{d}s_{\mathcal{T}}f_{\mathcal{T}}\left(\mathbf{p}_3,s_{\mathcal{T}}\right)\psi\left(\mathbf{p}_3,s-s_{\mathcal{T}}\right). \tag{8}$$

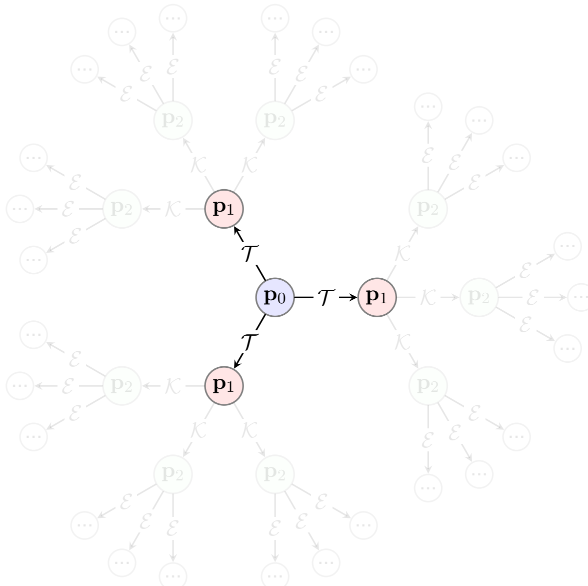
Forward Random Walk Interpretation ($\rightarrow$)

$$\psi_{\mathcal{E}}(\mathbf{p}_0, s) =$$
$$\mathcal{T}(\mathbf{p}_0, \mathbf{p}_1)$$
$$\times \mathcal{K}(\mathbf{p}_1, \mathbf{p}_2)$$
$$\times \mathcal{E}(\mathbf{p}_2, \mathbf{p}_3)$$
$$\times \int ds_{\mathcal{T}} f_{\mathcal{T}}(\mathbf{p}_3, s_{\mathcal{T}})$$
$$\times \psi(\mathbf{p}_3, s - s_{\mathcal{T}})$$



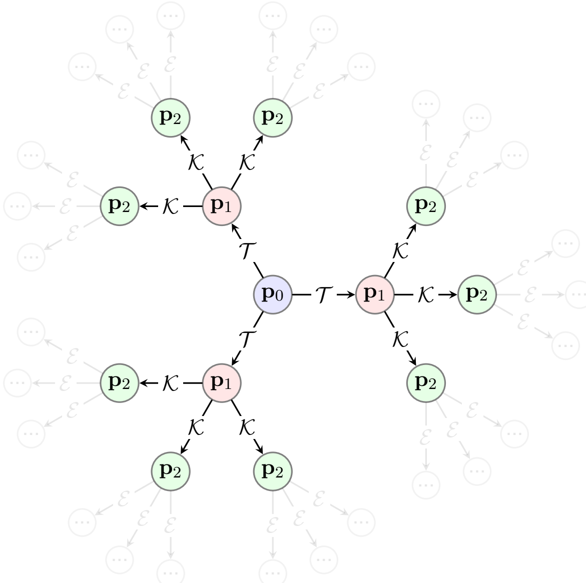
Forward Random Walk Interpretation ($\rightarrow$)

$$\psi_{\mathcal{E}}(\mathbf{p}_0, s) = \mathcal{T}(\mathbf{p}_0, \mathbf{p}_1) \times \mathcal{K}(\mathbf{p}_1, \mathbf{p}_2) \times \mathcal{E}(\mathbf{p}_2, \mathbf{p}_3) \times \int ds_{\mathcal{T}} f_{\mathcal{T}}(\mathbf{p}_3, s_{\mathcal{T}}) \times \psi(\mathbf{p}_3, s - s_{\mathcal{T}})$$



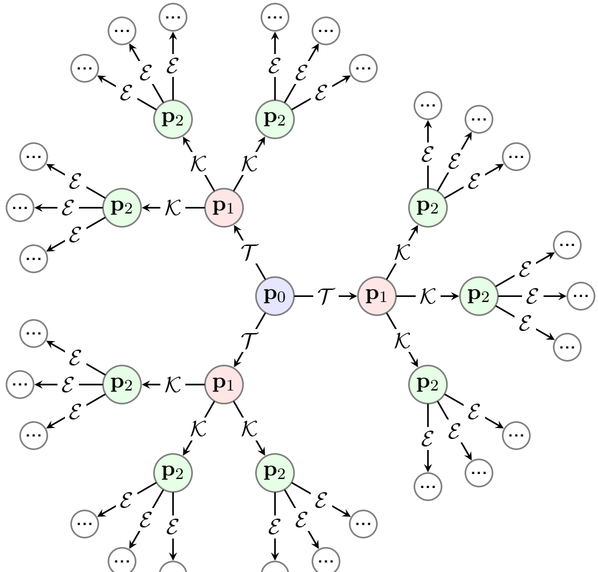
Forward Random Walk Interpretation ($\rightarrow$)

$$\psi_{\mathcal{E}}\left(\mathbf{p}_0, s\right) =$$
$$\mathcal{T}\left(\mathbf{p}_0, \mathbf{p}_1\right)$$
$$\times\mathcal{K}\left(\mathbf{p}_1, \mathbf{p}_2\right)$$
$$\times\mathcal{E}\left(\mathbf{p}_2, \mathbf{p}_3\right)$$
$$\times\int \mathrm{d}s_{\mathcal{T}} f_{\mathcal{T}}\left(\mathbf{p}_3, s_{\mathcal{T}}\right)$$
$$\times\psi\left(\mathbf{p}_3, s - s_{\mathcal{T}}\right)$$



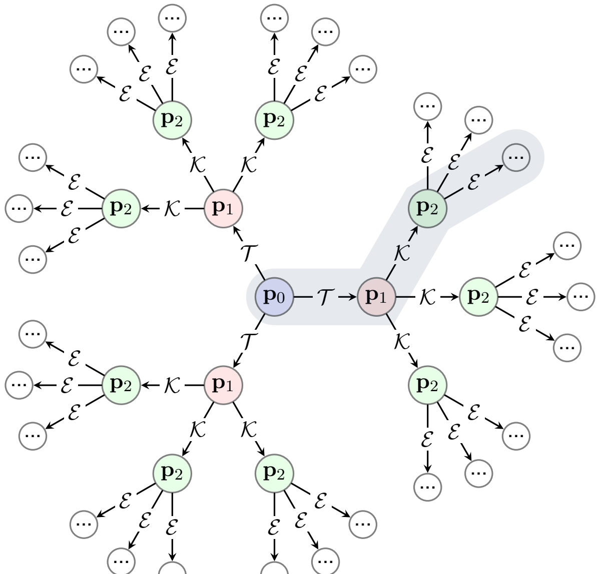
Forward Random Walk Interpretation ($\rightarrow$)

$$
\begin{aligned}
\psi_{\mathcal{E}}(\mathbf{p}_0, s) = \\
&\mathcal{T}(\mathbf{p}_0, \mathbf{p}_1) \\
&\times \mathcal{K}(\mathbf{p}_1, \mathbf{p}_2) \\
&\times \mathcal{E}(\mathbf{p}_2, \mathbf{p}_3) \\
&\times \int \mathrm{d}s_{\mathcal{T}} f_{\mathcal{T}}(\mathbf{p}_3, s_{\mathcal{T}}) \\
&\times \psi(\mathbf{p}_3, s - s_{\mathcal{T}})
\end{aligned}
$$



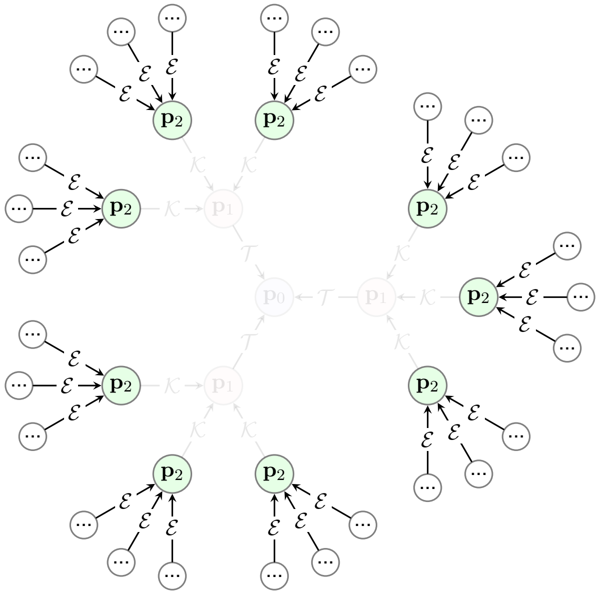
Forward Random Walk Interpretation ($\rightarrow$)

$$
\begin{aligned}
\psi_{\mathcal{E}}\left(\mathbf{p}_0, s\right) = \\
&\mathcal{T}\left(\mathbf{p}_0, \mathbf{p}_1\right) \\
&\times \mathcal{K}\left(\mathbf{p}_1, \mathbf{p}_2\right) \\
&\times \mathcal{E}\left(\mathbf{p}_2, \mathbf{p}_3\right) \\
&\times \int \mathrm{d}s_{\mathcal{T}} f_{\mathcal{T}}\left(\mathbf{p}_3, s_{\mathcal{T}}\right) \\
&\times \psi\left(\mathbf{p}_3, s - s_{\mathcal{T}}\right)
\end{aligned}
$$



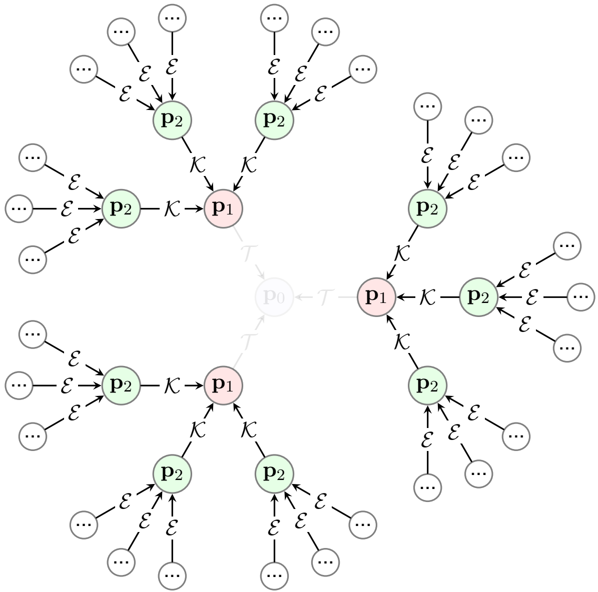
Adjoint Random Walk Interpretation ($\leftarrow$)

$$\psi_{\mathcal{E}}(\mathbf{p}_0, s) =$$
$$\mathcal{T}(\mathbf{p}_0, \mathbf{p}_1)$$
$$\times\mathcal{K}(\mathbf{p}_1, \mathbf{p}_2)$$
$$\times\mathcal{E}(\mathbf{p}_2, \mathbf{p}_3)$$
$$\times\int \mathrm{d}s_{\mathcal{T}} f_{\mathcal{T}}(\mathbf{p}_3, s_{\mathcal{T}})$$
$$\times\psi(\mathbf{p}_3, s - s_{\mathcal{T}})$$



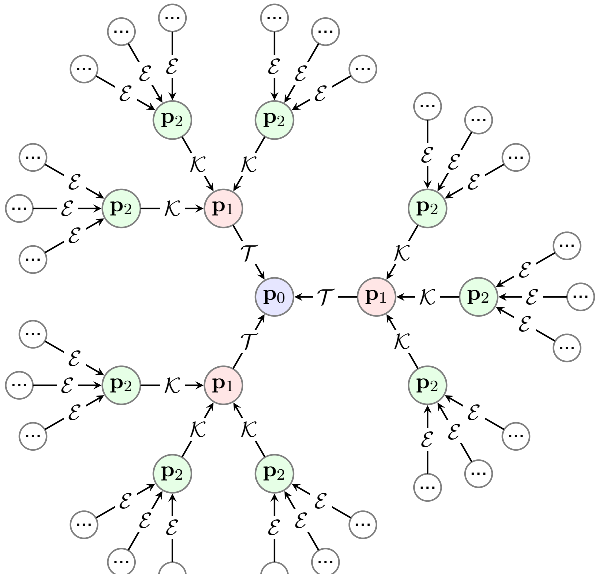
Adjoint Random Walk Interpretation (←)

$$\psi_{\mathcal{E}}\left(\mathbf{p}_0, s\right) =$$
$$\mathcal{T}\left(\mathbf{p}_0, \mathbf{p}_1\right)$$
$$\times \mathcal{K}\left(\mathbf{p}_1, \mathbf{p}_2\right)$$
$$\times \mathcal{E}\left(\mathbf{p}_2, \mathbf{p}_3\right)$$
$$\times \int \mathrm{d}s_{\mathcal{T}} f_{\mathcal{T}}\left(\mathbf{p}_3, s_{\mathcal{T}}\right)$$
$$\times \psi\left(\mathbf{p}_3, s - s_{\mathcal{T}}\right)$$



Adjoint Random Walk Interpretation (←)

$$\psi_{\mathcal{E}}\left(\mathbf{p}_0, s\right) = \mathcal{T}\left(\mathbf{p}_0, \mathbf{p}_1\right) \times \mathcal{K}\left(\mathbf{p}_1, \mathbf{p}_2\right) \times \mathcal{E}\left(\mathbf{p}_2, \mathbf{p}_3\right) \times \int \mathrm{d}s_{\mathcal{T}} f_{\mathcal{T}}\left(\mathbf{p}_3, s_{\mathcal{T}}\right) \times \psi\left(\mathbf{p}_3, s - s_{\mathcal{T}}\right)$$



HSME Calculation and Use

- The m moments of the history-score distribution are:

$$\Psi_m(\mathbf{p}_0) = \int_{-\infty}^{\infty} s^m \psi(\mathbf{p}_0, s)\, \mathrm{d}s \tag{9}$$

- For $m = 1$: comparable to adjoint integral transport equation
- Associated tally responses (with physical source term $Q(\mathbf{p}_0)$) are:

$$M_m = \int \Psi_m(\mathbf{p}_0)\, Q(\mathbf{p}_0)\, \mathrm{d}\mathbf{p}_0 \tag{10}$$

- Population variance is:

$$\sigma^2 = M_2 - M_1^2 \tag{11}$$

- Difficulty of solving for $\Psi_{m>1}(\mathbf{p}_0)$ affected by VR techniques used

Forced-flight Biasing & Transmission Kernels

- Forced-flight biasing kernel (for notational convenience):

$$\mathcal{B}_{\text{ff}}\left(\mathbf{p},\mathbf{p}'\right) \equiv \int \mathrm{d}V' \int \mathrm{d}\Omega' \int \mathrm{d}E' \int \mathrm{d}w'$$
$$\mathbb{1}\left(\mathbf{x} \notin \{\mathbf{x}_{\text{ff}}\}\right) \delta\left(\mathbf{x}' - \mathbf{x}_{\text{ff}}\left(\mathbf{x},\boldsymbol{\Omega}'\right)\right)$$
$$\times\, \tilde{g}\left(\boldsymbol{\Omega},E \to \boldsymbol{\Omega}',E'\right)$$
$$\times \left[\beta\left(\mathbf{x}\right)\delta\left(w' - \frac{w}{\beta\left(\mathbf{x}\right)}\frac{g\left(\boldsymbol{\Omega},E \to \boldsymbol{\Omega}',E'\right)}{\tilde{g}\left(\boldsymbol{\Omega},E \to \boldsymbol{\Omega}',E'\right)}\exp\left[-\lambda\left(\mathbf{x},\mathbf{x}_{\text{ff}},E\right)\right]\right)\right.$$
$$\left. +\left(1-\beta\left(\mathbf{x}\right)\right)\delta\left(w'-0\right)\right]$$

where

$\mathbb{1}\left(\mathbf{x} \notin \{\mathbf{x}_{\text{ff}}\}\right)$ indicator function whether original position is inside or on surface of forced-flight region

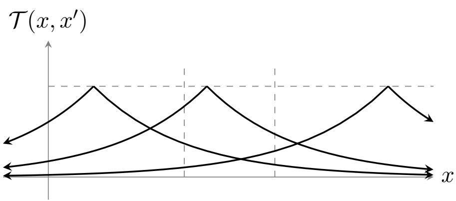
$\delta\left(\cdot\right)$ Dirac delta

$\mathbf{x}_{\text{ff}}\left(\mathbf{x},\boldsymbol{\Omega}'\right)$ nearest boundary point of forced flight to x along $\boldsymbol{\Omega}'$

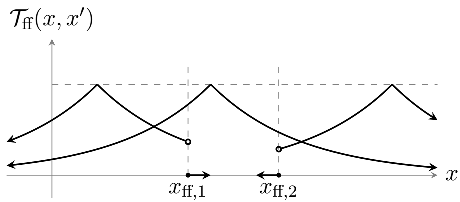
Forced-flight Biasing & Transmission Kernels

- Complementary 1-D forced-flight-associated free-flight transmission kernel

$$\mathcal{T}_{\mathrm{ff}}\left(\mathbf{p},\mathbf{p}'\right) = \begin{cases} 0, & \left[\begin{array}{c} \mathbf{x} \notin \{\mathbf{x}_{\mathrm{ff}}\} \text{ and} \\ \frac{\mathbf{x}'-\mathbf{x}}{\|\mathbf{x}'-\mathbf{x}\|} \cdot \frac{\mathbf{h}}{\|\mathbf{h}\|} = 1 \text{ and} \\ \|\mathbf{x}'-\mathbf{x}\| \geq \|\mathbf{h}\| \end{array}\right] \forall\, \mathbf{h} \in \{\mathbf{x}_{\mathrm{ff}} - \mathbf{x}\} \\ \mathcal{T}\left(\mathbf{p},\mathbf{p}'\right), & \text{otherwise} \end{cases} \tag{12}$$

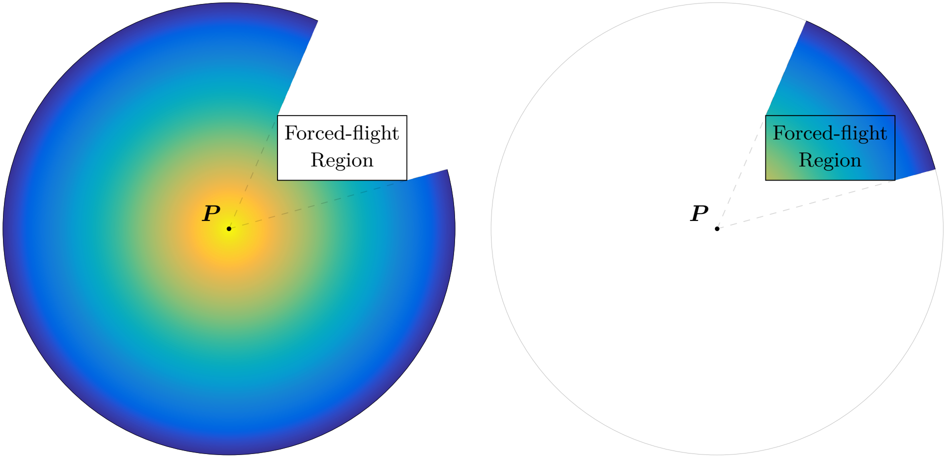


Analog



Forced Flight

Forced-flight Biasing & Transmission Kernels



Random Walk Incorporating Forced-flight

- The Monte Carlo random walk with forced-flight is:

$$\psi_{\mathrm{ff}}\left(\mathbf{p}_0, s\right) = \mathcal{T}_{\mathrm{ff}}\left(\mathbf{p}_0, \mathbf{p}_1\right) \mathcal{K}\left(\mathbf{p}_1, \mathbf{p}_2\right) \times \mathcal{E}\left(\mathbf{p}_2, \mathbf{p}_3\right) \int \mathrm{d}s_{\mathcal{T}} f_{\mathcal{T}}\left(\mathbf{p}_3, s_{\mathcal{T}}\right) \int \mathrm{d}s_{\mathcal{E}}\, \psi\left(\mathbf{p}_3, s_{\mathcal{E}}\right) \times \mathcal{B}_{\mathrm{ff}}\left(\mathbf{p}_2, \mathbf{p}_4\right) \psi\left(\mathbf{p}_4, s - s_{\mathcal{T}} - s_{\mathcal{E}}\right), \tag{13}$$

- Forced-flight acts as a conditional splitting process
  - Applied in concert with analog non-absorptive collision and surface crossing
- Truncating transmission kernel applies for all random walks

Forced-flight Second HSME Solution Approach

- Operate on both sides as $\int \mathrm{d}s\, s^2 \left(\cdot\right)$
- Substitute $u = s - s_{\mathcal{T}}$ so $s = u + s_{\mathcal{T}}$ and $\mathrm{d}s = \mathrm{d}u$ to obtain

$$\begin{aligned}\Psi_{2,\mathrm{ff}}\left(\mathbf{p}_0\right) = \mathcal{T}_{\mathrm{ff}}\left(\mathbf{p}_0, \mathbf{p}_1\right) \mathcal{K}\left(\mathbf{p}_1, \mathbf{p}_2\right) \\ \times \mathcal{E}\left(\mathbf{p}_2, \mathbf{p}_3\right) \int \mathrm{d}s_{\mathcal{T}}\, f_{\mathcal{T}}\left(\mathbf{p}_3, s_{\mathcal{T}}\right) \int \mathrm{d}s_{\mathcal{E}}\, \psi\left(\mathbf{p}_3, s_{\mathcal{E}}\right) \\ \times \mathcal{B}_{\mathrm{ff}}\left(\mathbf{p}_2, \mathbf{p}_4\right) \int \mathrm{d}u\, \left(u + s_{\mathcal{E}} + s_{\mathcal{T}}\right)^2 \psi\left(\mathbf{p}_4, u\right). \end{aligned} \quad (14)$$

- Distribute the binomial and perform the integrations to obtain moments
- Separate into $\Psi_{2,\mathrm{ff}}$ and Q_2 to ease incorporation into solver
  - Result on next slide

Forced-flight Second HSME Result

$$\begin{aligned}\Psi_{2,\text{ff}}(\mathbf{p}_0) &= \mathcal{T}_{\text{ff}}(\mathbf{p}_0,\mathbf{p}_1)\,\mathcal{K}(\mathbf{p}_1,\mathbf{p}_2)\,\mathcal{E}(\mathbf{p}_2,\mathbf{p}_3)\,\Psi_2(\mathbf{p}_3)\\ &+\mathcal{T}_{\text{ff}}(\mathbf{p}_0,\mathbf{p}_1)\,\mathcal{K}(\mathbf{p}_1,\mathbf{p}_2)\,\mathcal{B}_{\text{ff}}(\mathbf{p}_2,\mathbf{p}_4)\Psi_2(\mathbf{p}_4)\\ &+Q_{2,\text{ff}}(\mathbf{p}_0)\end{aligned} \tag{15a}$$

$$\begin{aligned}Q_{2,\text{ff}}(\mathbf{p}_0) &= \mathcal{T}_{\text{ff}}(\mathbf{p}_0,\mathbf{p}_1)\,\mathcal{K}(\mathbf{p}_1,\mathbf{p}_2)\,\mathcal{E}(\mathbf{p}_2,\mathbf{p}_3)\,\overline{s^2}_{\mathcal{T}}(\mathbf{p}_3)\\ &+2\mathcal{T}_{\text{ff}}(\mathbf{p}_0,\mathbf{p}_1)\,\mathcal{K}(\mathbf{p}_1,\mathbf{p}_2)\,\mathcal{E}(\mathbf{p}_2,\mathbf{p}_3)\,\Psi_1(\mathbf{p}_3)\,\overline{s}_{\mathcal{T}}(\mathbf{p}_3)\\ &+2\mathcal{T}_{\text{ff}}(\mathbf{p}_0,\mathbf{p}_1)\,\mathcal{K}(\mathbf{p}_1,\mathbf{p}_2)\,\mathcal{B}_{\text{ff}}(\mathbf{p}_2,\mathbf{p}_4)\overline{s}_{\mathcal{T}}(\mathbf{p}_3)\,\Psi_1(\mathbf{p}_4)\\ &+2\mathcal{T}_{\text{ff}}(\mathbf{p}_0,\mathbf{p}_1)\,\mathcal{K}(\mathbf{p}_1,\mathbf{p}_2)\,\underbrace{\mathcal{E}(\mathbf{p}_2,\mathbf{p}_3)\,\Psi_1(\mathbf{p}_3)\,\mathcal{B}_{\text{ff}}(\mathbf{p}_2,\mathbf{p}_4)\Psi_1(\mathbf{p}_4)}_{\text{Cross term}}\end{aligned} \tag{15b}$$

Future Time Equation (FTE)

- Each event e requires some processing time τ_e, which is represented by a time "scoring" function

$$f_e(\mathbf{p}, \tau) = \delta(\tau - \tau_e) \tag{16}$$

from the events

τ_{tally} time to process a tally event,
τ_{col} time to process a collision event,
τ_{xs} time to process a cross-section lookup,
τ_{geom} time to perform transmission along a free-flight trajectory,
τ_{rt} time to perform ray tracing, per Monte Carlo surface, for forced-flight variance reduction,
τ_{ff} time to perform all other forced-flight variance reduction processing,
τ_{surf} time to process surface-crossing,
τ_{bank} time to process a particle bank event, and
τ_{src} time to process a forward source event as noted previously.

Future Time Equation (FTE), cont.

- Construct FTPDF

$$\upsilon\left(\mathbf{p}_0,\tau\right)=\upsilon_{\mathcal{S}}\left(\mathbf{p}_0,\tau\right)+\upsilon_{\text{ff}}\left(\mathbf{p}_0,\tau\right)+\upsilon_{\mathcal{A}}\left(\mathbf{p}_0,\upsilon\right) \tag{17}$$

compared with the analog FTPDF

$$\upsilon\left(\mathbf{p}_0,\tau\right)=\upsilon_{\mathcal{S}}\left(\mathbf{p}_0,\tau\right)+\upsilon_{\mathcal{E}}\left(\mathbf{p}_0,\tau\right)+\upsilon_{\mathcal{A}}\left(\mathbf{p}_0,\upsilon\right) \tag{18}$$

- Integrate to get the first moment, the future-time equation (FTE)

$$\Upsilon\left(\mathbf{p}_0\right)=\int \mathrm{d}\tau\,\tau\,\upsilon\left(\mathbf{p}_0,\tau\right) \tag{19}$$

- The partial forced-flight biased FTE is:

$$\begin{aligned}\Upsilon_{\text{ff}}\left(\mathbf{p}_0\right)=\mathcal{T}_{\text{ff}}\left(\mathbf{p}_0,\mathbf{p}_1\right)\{&\overline{\tau_{\text{geom}}}\left(\mathbf{p}_2\right)+\mathcal{K}\left(\mathbf{p}_1,\mathbf{p}_2\right)[\overline{\tau_{\text{col}}}\left(\mathbf{p}_2\right)\\&+\mathcal{B}_{\text{ff}}\left(\mathbf{p}_2,\mathbf{p}_4\right)[\beta\left(\mathbf{p}_4\right)\overline{\tau_{\text{ff}}}\left(\mathbf{p}_4\right)+\beta\left(\mathbf{p}_4\right)n_{\text{surf}}\left(\mathbf{p}_4\right)\overline{\tau_{\text{rt}}}\left(\mathbf{p}_4\right)+\Upsilon\left(\mathbf{P}_4\right)]\\&+\mathcal{E}\left(\mathbf{p}_2,\mathbf{p}_3\right)[\overline{\tau_{\text{tally}}}\left(\mathbf{p}_3\right)+\overline{\tau_{\text{xs}}}\left(\mathbf{p}_3\right)+\Upsilon\left(\mathbf{P}_3\right)]]\}\,.\end{aligned} \tag{20}$$

Summary & Future Work

- History-score moment equations
  - Biasing kernel deduced, HSPDF constructed, HSMEs derived
  - Show reasonable moment-ordered quantities and cross terms
- Future time scales according to
  - Local forced flight rouletting parameter, β
  - Number of Monte Carlo surfaces along forced-flight trajectory
- Future work:
  - Apply forced-flight variance reduction to source-emission events
  - Analyze multiple forced-flight regions
  - Incorporate forced-flight weight cutoffs

Questions?

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Backup Slides

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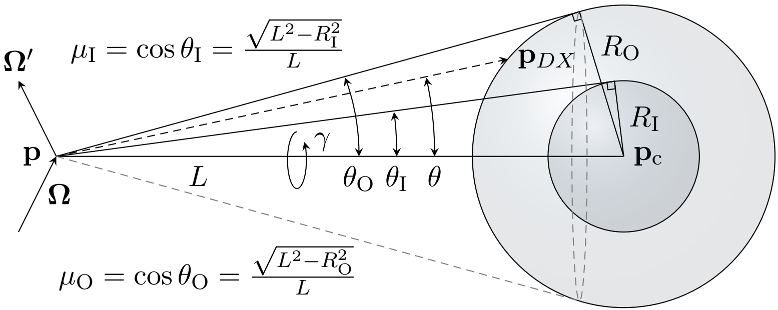
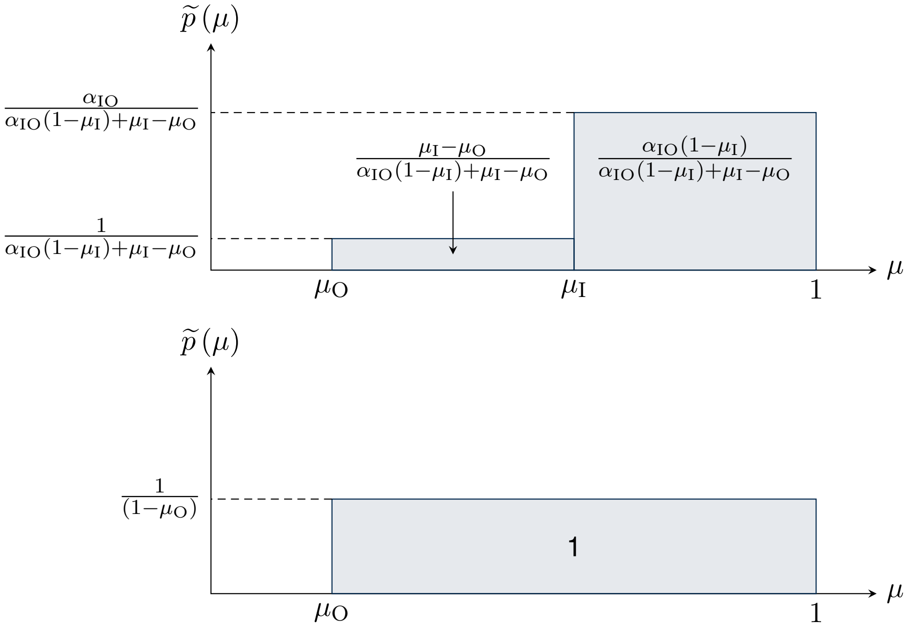
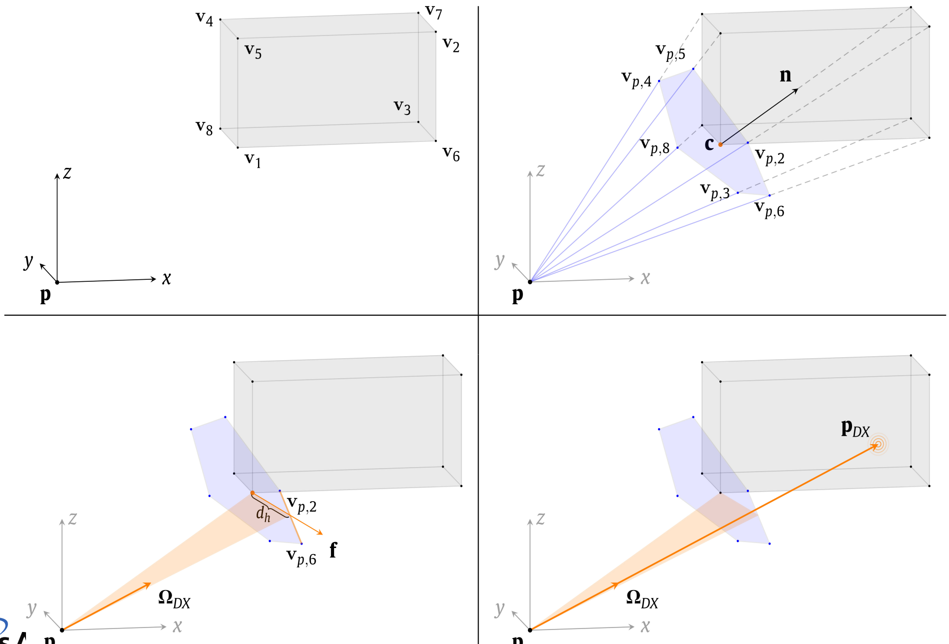
Full DXTRAN Treatment



Inner/Outer Sphere Polar Cosine Biasing PDF



Arbitrary Convex Polyhedral DXTRAN Process



CADIS & FW-CADIS Overview

Adjoint Solution

Forward flux, $\mathbf{M}^*\psi^* = \Sigma_d$

Total response, $R = \langle q, \psi^* \rangle$

Forward Solution

Forward flux, $\mathbf{M}\psi = q$

Total response, $R = \langle \Sigma_d, \psi \rangle$

CADIS

Adjoint flux, $\mathbf{M}^*\psi^* = \Sigma_d$

Total response, $R = \langle q, \psi^* \rangle$

Weight targets, $w = R/\psi^*$

Biased source, $\tilde{q} = \psi^* q/R$

FW-CADIS

Forward flux, $\mathbf{M}\psi = q$

Adjoint flux, $\mathbf{M}^*\psi^* = \Sigma_d / \langle \Sigma_d, \psi \rangle_{E,\mathbf{\Omega}}$

Total response, $R = \langle q, \psi^* \rangle$

Weight targets, $w = R/\psi^*$

Biased source, $\tilde{q} = \psi^* q/R$

History-Score-Moment Equation Weight Separability

- Weight separability[1] with weight-independent VR techniques:

$$\Psi_m(\mathbf{R}, aw) = a^m \Psi_m(\mathbf{R}, w) = a^m w^m \Psi_m(\mathbf{R}, w = 1) \tag{21}$$

- This separability shows that weight-independent techniques do not require a discretized weight mesh for any moments
  - Reduced memory requirements
  - Reduced deterministic solver computational time
  - Permits easier incorporation into pre-existing deterministic solver

[1] T. E. Booth et al., Nucl. Sci. & Eng., vol. 71, pp. 128–142, Aug. 1979.