LA-UR-14-24365

Title:	Comparison of Prompt Kinetics Models Derived from Alternate Eigenvalues
Author(s):	Kiedrowski, Brian C.
Intended for:	American Nuclear Society Annual Meeting 2014, 2014-06-15/2014-06-19 (Anaheim, California, United States)
Issued:	2014-06-13

Comparison of Prompt Kinetics Models Derived from Alternate Eigenvalues

Brian C. Kiedrowski

Los Alamos National Laboratory

June 17, 2014

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Abstract

The classic point kinetics model is typically derived using the time-dependent neutron transport equation and a static-*k* eigenvalue form of the adjoint transport equation with a respective forward shape function. This approximation is valid near criticality and can be used to estimate the inverse prompt period α; the accuracy degrades for systems far from criticality. In principle, the α eigenvalue equation can be solved in this regime, but the stiffness of the equations for subcritical systems makes this numerically difficult to solve. Therefore, formulating the point kinetics equations using collision *c* and leakage *l* eigenvalues are studied to understand if the domain of applicability can be extended.

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Introduction

- Motivation
- Theory
- Results

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Motivation

- Measurements of the inverse prompt period α are sometimes made on subcritical assemblies.
- Point kinetics models may be used to estimate α if the system is near critical.
- Question: Can using different static-eigenvalue equations extend the range of applicability of point kinetics?

Time-Dependent Neutron Transport

- Neutron behavior described by time-dependent transport equation:

$$\frac{1}{v}\frac{\partial\psi}{\partial t} = (S + M - L - T)\psi(\mathbf{r}, \hat{\mathbf{\Omega}}, E, t) + \sum_i \lambda_i C_i(t) + Q(\mathbf{r}, \hat{\mathbf{\Omega}}, E, t),$$

- Asymptotic prompt behavior may be obtained by solving the α eigenvalue problem:

$$(S + M - L - T)\,\psi_\alpha = \frac{\alpha}{v}\psi_\alpha.$$

- Numerically difficult to solve for subcritical systems.

Point Kinetics Model

- In the case where the system is not too subcritical, point kinetics may be used to approximate the prompt α.
- Typically assumes a k eigenvalue form of the transport equation to obtain the shape and the adjoint weighting functions.

$$(L + T - S)\,\psi_k = \frac{1}{k}\,(M + B)\,\psi_k.$$

- The use of k is convenient for many reasons, including the fact that many standard transport software packages support its calculation.
- Not the only multiplicative static eigenvalue that could be solved.
- Other choices may yield a model that gives a better approximation for α further from criticality.

Generalized Multiplicative Eigenvalue Formulation

- The k eigenvalue is one of many possible multiplicative eigenvalues.
- Define the generalized multiplicative eigenvalue x:

$$H_x \psi_x = \frac{1}{x} \left(G_x + B \right) \psi_x .$$

- Here B is the delayed neutron emission operator, and H_x and G_x are operators for the left- and right-hand sides of the transport equation, depending upon the choice of x.
- E.g., for $x = k$, $H_x = L + T - X$ and $G_x = M$.
- Criticality condition is $x = 1$, regardless of choice of x.

Generalized Eigenvalue Point Kinetics

- Starting with time-dependent neutron transport equation, assume fundamental mode and separability of time:

$$\psi(\mathbf{r}, \hat{\mathbf{\Omega}}, E, t) = n(t)\varphi(\mathbf{r}, \hat{\mathbf{\Omega}}, E).$$

- Multiply by adjoint function $\psi_x^\dagger$, integrate over all space.
- Take corresponding adjoint function, multiply by forward function ψ_x, integrate over all space.
- Subtract the two and rearrange.

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Generalized Eigenvalue Point Kinetics

- The following equation is obtained for the time-rate of change of the neutron level $n(t)$:

$$\frac{dn}{dt} = \frac{\frac{x-1}{x}\left\langle \psi_x^\dagger, \left(G_x + B\right)\varphi \right\rangle - \left\langle \psi_x^\dagger, B\varphi \right\rangle}{\left\langle \psi_x^\dagger, \frac{1}{v}\varphi \right\rangle} n(t) + \sum_i \frac{\left\langle \psi_x^\dagger, \lambda_i C_i(t) \right\rangle}{\left\langle \psi_x^\dagger, \frac{1}{v}\varphi \right\rangle} + \frac{\left\langle \psi_x^\dagger, Q(t) \right\rangle}{\left\langle \psi_x^\dagger, \frac{1}{v}\varphi \right\rangle}.$$

- Next make some convenient definitions.

Generalized Eigenvalue Point Kinetics

- Reactivity:

$$\rho_x = \frac{x-1}{x};$$

- Effective reproduction time:

$$\Lambda_x = \frac{\left\langle \psi_x^\dagger, \frac{1}{v}\varphi \right\rangle}{\left\langle \psi_x^\dagger, \left(G_x + B\right)\varphi \right\rangle};$$

- Effective delayed neutron fraction:

$$\beta_x = \frac{\left\langle \psi_x^\dagger, B\varphi \right\rangle}{\left\langle \psi_x^\dagger, \left(G_x + B\right)\varphi \right\rangle};$$

Generalized Eigenvalue Point Kinetics

- Effective ith species precursor concentration:

$$c_{x,i}(t) = \frac{\left\langle \psi_x^\dagger, \lambda_i C_i(t) \right\rangle}{\left\langle \psi_x^\dagger, \frac{1}{v}\varphi \right\rangle};$$

- Effective ith species precursor concentration:

$$\beta_{x,i} = \frac{\left\langle \psi_x^\dagger, B_i \varphi \right\rangle}{\left\langle \psi_x^\dagger, (G_x + B)\, \varphi \right\rangle};$$

- Effective source term:

$$q_x(t) = \frac{\left\langle \psi_x^\dagger, Q(t) \right\rangle}{\left\langle \psi_x^\dagger, \frac{1}{v}\varphi \right\rangle}.$$

Generalized Eigenvalue Point Kinetics

- Derive a familiar form:

$$\frac{dn}{dt} = \frac{\rho_x - \beta_x}{\Lambda_x} n(t) + \sum_i \lambda_i c_{x,i}(t) + q_x(t),$$

$$\frac{dc_{x,i}}{dt} = \frac{\beta_{x,i}}{\Lambda_x} n(t) - \lambda_i c_{x,i}(t).$$

- The point kinetics asymptotic inverse prompt period is:

$$\alpha_x = \frac{\rho_x - \beta_x}{\Lambda_x}.$$

Three Specific Cases:

- Multiplication eigenvalue:

$$(L + T - S)\,\psi_k = \frac{1}{k}\,(M + B)\,\psi_k;$$

- Collision eigenvalue:

$$(L + T)\,\psi_c = \frac{1}{c}\,(S + M + B)\,\psi_c;$$

- Leakage eigenvalue:

$$L\psi_l = \frac{1}{l}\,(S + M + B - T)\,\psi_l.$$

Results

- Implemented in SN (discrete ordinates) as proof of concept.
- Three multigroup slab test problems:
  - Bare, fast (4-group), vary slab thickness
  - Reflected, fast (4-group), vary reflector thickness
  - Reflected, thermal (8-group), vary fuel/moderator ratio.
- Use S_{64} Gauss-Legendre quadature, 1000 total spatial elements.
- Compare results with Monte Carlo calculation of asymptotic α obtained directly from slope of logarithm of neutron population:

$$\alpha(t) = \frac{1}{N(t)} \frac{dN}{dt}, \quad t \to \infty.$$

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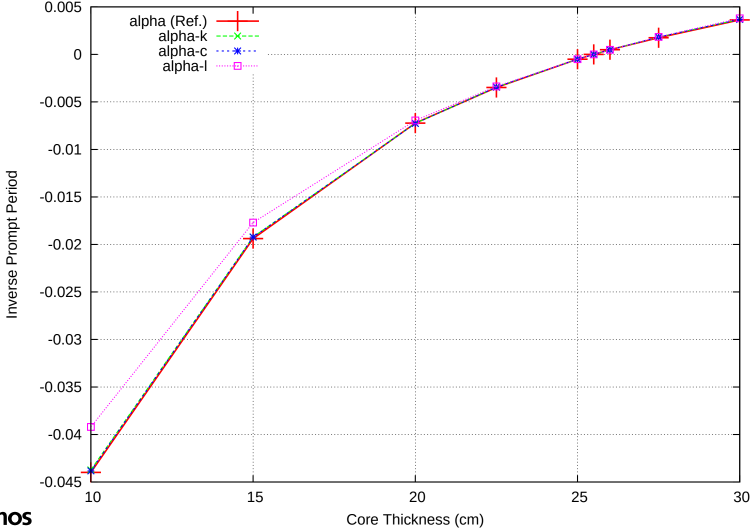
SN, Bare-Fast Case



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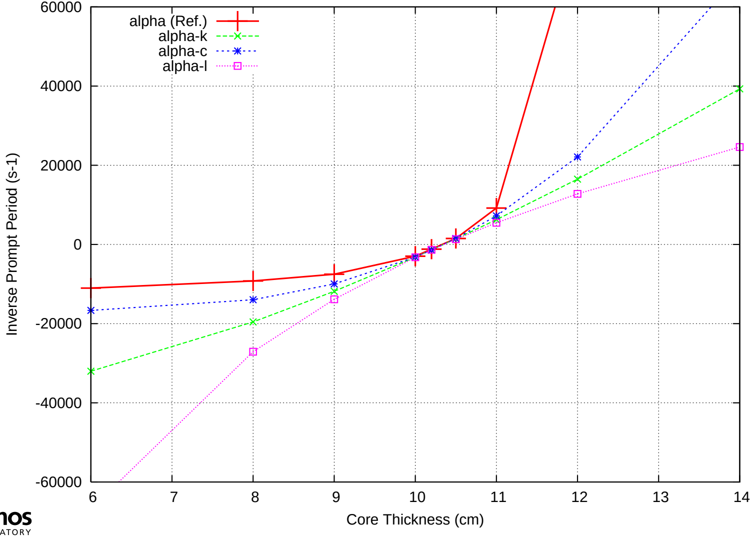
SN, Reflected-Fast Case



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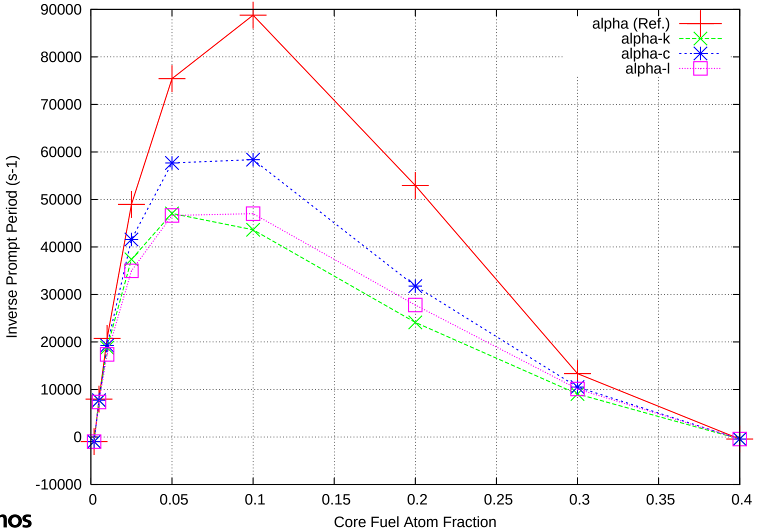
SN, Reflected-Thermal Case



Discussion

- For these simple test cases, results indicate that using a point-kinetics model derived from the c eigenvalue may provide a more widely applicable corresponding inverse prompt period α_c as a surrogate for α.
- These conclusions are tenuous and really need to be assessed versus more complicated problems.
- Next step: Continuous-energy Monte Carlo implementation.

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Future Work: CE Monte Carlo

- First, a continuous-energy Monte Carlo code needs to be adapted to calculate generalized eigenvalues (at least c and l).
- Secondly, adjoint weighted tallies need to be incorporated into the software (e.g., MCNP has this for k).
- Iterated probability methods can likely be extended: adjoint function as the expected population numerous generations later because of a hypothetical neutron at each point.
- MCNP research prototype exists for the c eigenvalue, but incomplete right now.

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Acknowledgments

- Funding provided by the U.S. DOE/NNSA Advanced Scientific Computing Program and Nuclear Criticality Safety Program.

Questions?