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Title: Evaluation of Computing c-Eigenvalues with Monte Carlo

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Evaluation of Computing c-Eigenvalues with Monte Carlo

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Abstract

Three different eigenvalues of the transport equations, and their advantages and disadvantages are discussed. A method for computing the collision or c eigenvalue is explained. Computing c rather than k generally takes longer to converge, but also has a higher figure of merit during active cycles, suggesting an advantage for use in criticality or α-eigenvalue searches.

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Roadmap

- A Chat on Eigenvalues
- c-Eigenvalue Power Iteration
- Results & Efficiency Studies
- Outstanding Issues

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Why Pose as an Eigenvalue Problem?

- Typically want static or asymptotic behavior.
- Understand subcritical versus critical versus supercritical.

- Eigenvalues function as a “knob” to balance sources and losses.
- Infinitely many possible for this purpose.
- All identical at criticality.

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Why k? A Difficult Question...

$$L\psi + T\psi = S\psi + \frac{1}{k}F\psi$$

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What About α?

$$L\psi + T\psi + \frac{\alpha}{v}\psi = S\psi + F\psi$$

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How About the Collisional or c-Eigenvalue?

$$L\psi + T\psi = \frac{1}{c}\left(S + F\right)\psi$$

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Monte Carlo – The Power Iteration for k

- One equations and two unknowns (k, ψ); must solve iteratively.
- Guess k^0 and $F\psi^0$.

1. Simulate one fission generation of neutrons from $F\psi^n$.
2. At each collision, estimate k^{n+1} by $w\frac{\nu\Sigma_f}{\Sigma_t}$,
3. and new $F\psi^{n+1}$ by banking $\left\lfloor \frac{w}{k^n}\frac{\nu\Sigma_f}{\Sigma_t} + \xi \right\rfloor$ neutrons.
4. Renormalize $F\psi^{n+1}$.

- Repeat 1-4 until convergence.

The Power Iteration for c

- Define the total secondary production rate:

$$\Sigma_P = \nu\Sigma_f + \sum_{x=1}^{\infty} x\Sigma_{\mathrm{n},x\mathrm{n}}$$

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The Power Iteration for c

- One equations and two unknowns (c, ψ); must solve iteratively.
- Guess c^0 and $(S+F)\psi^0$.

1. Simulate one fission generation of neutrons from $(S+F)\psi^n$.
2. At each collision, estimate c^{n+1} by $w\frac{\Sigma_P}{\Sigma_t}$,
3. and new $(S+F)\psi^{n+1}$ by banking $\left\lfloor \frac{w}{c^n}\frac{\Sigma_P}{\Sigma_t}+\xi \right\rfloor$ neutrons.
4. Renormalize $(S+F)\psi^{n+1}$.

- Repeat 1-4 until convergence.

Does it Work?

- Two simple test problems:
  One group Pu-239 sphere, $\alpha = 0$.
  Two group infinite medium, $\alpha = (7 + \sqrt{145})/24 \approx 0.79340$.

- Conclusion: Both agree within *five digits of accuracy*.

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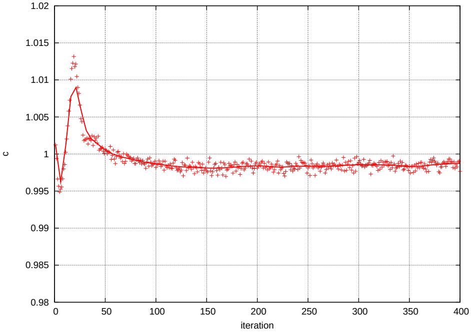
What About Convergence?

- 3 × 2 array of cans of Pu-nitrate solution:



- Takes longer to converge than k.

Test Problem Without Fissionable Material

- c eigenvalue defined for any system where scattering or fission possible.
- Test case: 5 cm diameter sphere of water.

- Spectrum exhibits Maxwellian shape at thermal energies.
- Free-gas treatment of hydrogen: $c = 0.84430$.
- $S(\alpha, \beta)$ hydrogen scattering: $c = 0.94568$.

Efficiency Measures

- Longer convergence, faster cycles.
- How to define measure of "goodness".

- Figure of Merit (active cycles only):

$$FOM = \frac{1}{R^2 \tau}$$

Define G as ratio of FOM for c to k.

- Inactive cycle wall-clock time W for convergence (trend in k or c used).

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Test Problems

1. Be-reflected sphere of HEU.
2. 3×2 array of cans of Pu-nitrate solution.
3. 3-D full-core PWR (Hoogenboom-Martin benchmark).

- Batch size of 10,000.
- Number of active cycles is 200.
- Number of inactive based on trend in k or c.

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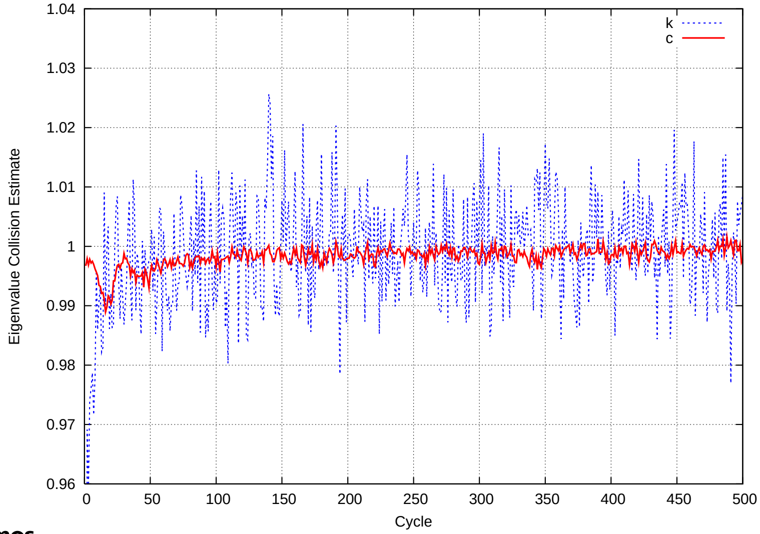
Efficiency Results

Table: Performance data for three test cases.

case	k	W_k	c	W_c	G
1	0.9955(4)	0.3	0.9954(3)	0.1	30
2	0.9866(7)	0.8	0.9989(1)	0.6	60
3	0.9992(5)	3.4	0.9986(1)	1.9	200

Efficiency Results (Full-Core PWR)



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Outstanding Issues

- Are these efficiency gains real? Inter-cycle correlation effects.
- Can convergence be accelerated?
- Other issues: Bias? Spectral effects? ...?

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Future Prospects

- In-line criticality search. May be more efficient with c.
- Some evidence to suggest α-eigenvalue iterations are easier and more efficient with c.

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Questions?