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Adjoint Weighting for Critical Systems with Continuous Energy Monte Carlo

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Abstract

Adjoint weighting is important for calculating parameters in reactor physics. A New method is developed to calculate adjoint-weighted quantities with continuous energy Monte Carlo. The method is applied to computing point-reactor kinetics parameters and estimating changes in reactivity from small perturbations. The results are benchmarked to 1D discrete ordinates, experimental data, and direct Monte Carlo calculations.

Outline

- Need for adjoint-weighting in Monte Carlo
- Method description
- Reactor kinetics
- Perturbation theory

Background

Monte Carlo Methods

- **Direct simulation of radiation transport**
- **Advantages**
  - High-fidelity geometric representation
  - Continuous energy and angle physics
- **Weaknesses**
  - Slow statistical convergence compared to discrete ordinates
  - **Adjoint calculations difficult**

Why are adjoint fluxes useful?

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- **Many reactor physics quantities are ratios of weighted integrals (kinetics parameters, etc.)**

$$\Lambda = \frac{\left\langle \psi^{\dagger} \, {}^{1}\!/_{v} \, \psi \right\rangle}{\left\langle \psi^{\dagger} \mathbf{F} \psi \right\rangle}$$

- **Adjoint flux is convenient weighting factor**

$$\left\langle \psi^{\dagger} \mathbf{A} \psi \right\rangle = \left\langle \psi \mathbf{A}^{\dagger} \psi^{\dagger} \right\rangle$$

- **Adjoint flux corresponds to importance of radiation with respect to a “response function”**

Computing Adjoints

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- **Deterministic method process:**
  1. Invert sign of streaming operator
  2. Transpose scattering matrix
  3. Use standard solution techniques

- **Monte Carlo options:**
  – Invert radiation transport physics – very difficult in CE
  – Forward solution methods (weight window generator)

Response in Eigenvalue Problems

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- **What is the response function for the k-eigenvalue transport equation?**

$$\mathbf{H}^{\dagger}\psi^{\dagger} = \frac{1}{k}\mathbf{F}^{\dagger}\psi^{\dagger}$$

- **Iterated Fission Probability:**

Consider a neutron introduced into a critical system at a location in phase space. The expected steady state neutron population resulting from that original progenitor neutron is defined as the iterated fission probability.

Iterated Fission Probability

- Monte Carlo already follows a neutron and its progeny through successive generations
- Need to track information about its progenitor
- Can existing random walks of a *k*-eigenvalue calculation be used?

Related Work

- KENO eigenvalue contributon estimator
- MCNIC
- Nauchi & Kameyama reactor kinetics parameter calculations

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Method Overview

Method Terminology

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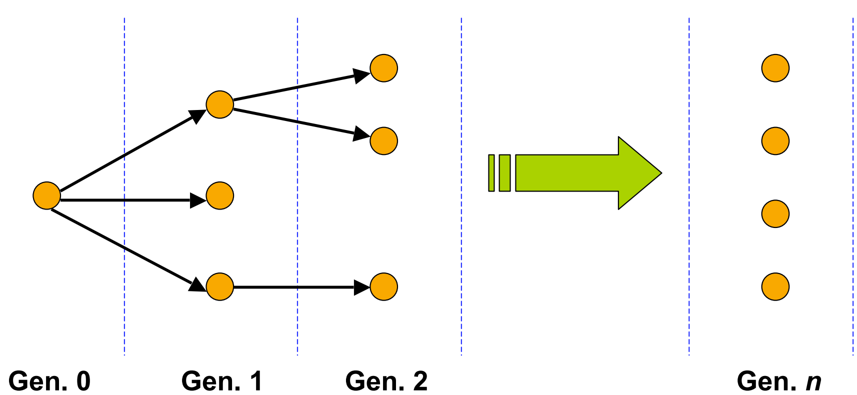
- Track neutron lineage through many generations.



Method Terminology

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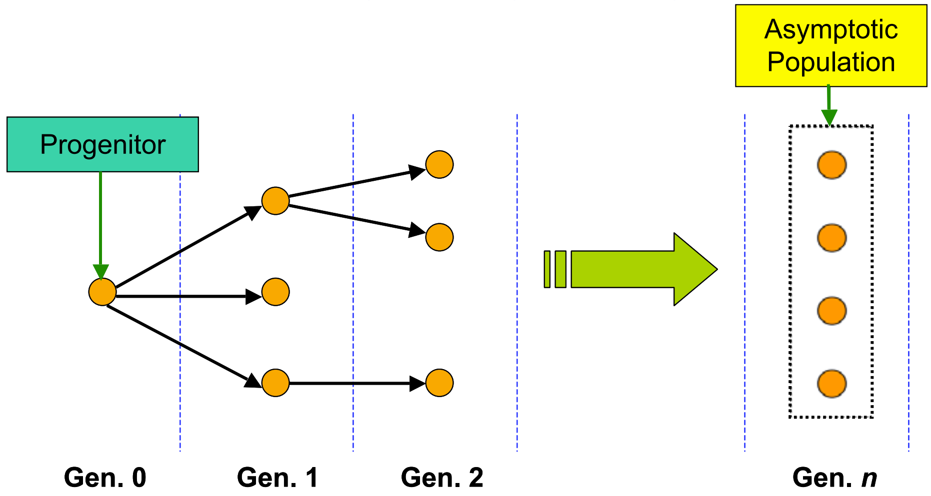

Method Terminology

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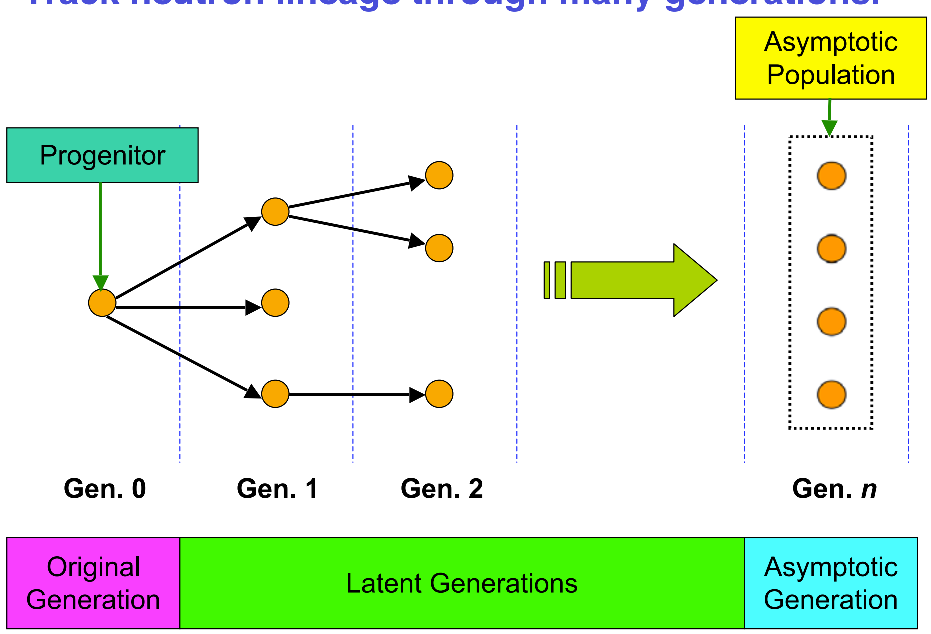
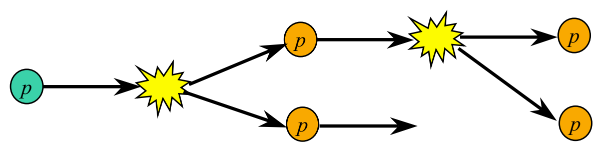


Method Overview

- **In original generation store contribution for each progenitor of index *p*:**
  - Ex. Fission Source Contribution: $\omega_p = w_{0,p}$
  - Ex. Track length flux: $\omega_p = \sum_{\tau \in p} w_{0,p} d_\tau$
- **Progenitor index *p* inherited by all progeny:**



- **Tally asymptotic population in asymptotic generation:**

$$\pi_p = \sum_{\tau \in p} \nu \Sigma_f w d_\tau \qquad \text{Tally} = \sum_p \pi_p \omega_p$$

Reactor Kinetics

Reactor Kinetics

- Point reactor approximation for criticality excursion analysis:

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n(t) + \sum_i \lambda_i C_i(t)$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i(t)$$

- Need to know averaged point reactor parameters for a given system.

Reactor Kinetics Parameters

- Neutron generation time:

$$\Lambda = \frac{\left\langle \psi^{\dagger}\, {}^{1}\!/_{v}\, \psi \right\rangle}{\left\langle \psi^{\dagger} \mathbf{F} \psi \right\rangle}$$

- Effective delayed neutron fraction:

$$\beta_{\text{eff}} = \frac{\left\langle \psi^{\dagger} \mathbf{B} \psi \right\rangle}{\left\langle \psi^{\dagger} \mathbf{F} \psi \right\rangle}$$

- Rossi-alpha:

$$\alpha = -\frac{\left\langle \psi^{\dagger} \mathbf{B} \psi \right\rangle}{\left\langle \psi^{\dagger}\, {}^{1}\!/_{v}\, \psi \right\rangle}$$

Reactor Kinetics Tallies

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- **Adjoint-weighted neutron density:**

$$\left\langle \psi^{\dagger} \frac{1}{v} \psi \right\rangle = \frac{1}{W} \sum_{p} \pi_p \sum_{\tau \in p} \frac{1}{v_\tau} w_{0,p} d_\tau$$

- **Adjoint-weighted total fission source:**

$$\left\langle \psi^{\dagger} \mathbf{F} \psi \right\rangle = \frac{1}{W} k \sum_{p} \pi_p w_{0,p}$$

- **Adjoint-weighted delayed fission source:**

$$\left\langle \psi^{\dagger} \mathbf{B} \psi \right\rangle = \frac{1}{W} k \sum_{p \in \beta} \pi_p w_{0,p}$$

Multigroup Partisn

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Lifetime Comparisons

#	G	Problem Description
1	4	Bare thermal slab, fuel/moderator mix
2	4	Reflected thermal slab, fuel + moderator
3	4	Bare fast slab
4	4	Reflected fast slab
5	8	Bare slab w/ intermediate spectrum
6	4	Bare fast sphere
7	4	Reflected fast sphere
8	4	Highly reflective slab
9	4	Subcritical bare fast slab (k = 0.78)
10	4	Supercritical bare fast slab (k = 1.14)

Multigroup Partisn

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Lifetime Comparisons

#	Partisn	MCNP
1	14.1323 µs	14.1025 +/- 0.0545 µs
2	135.2317 µs	135.0876 +/- 0.2081 µs
3	9.8100 ns	9.8099 +/- 0.0010 ns
4	43.4114 ns	43.5719 +/- 0.0913 ns
5	112.0523 ns	112.5003 +/- 0.4341 ns
6	1.7211 ns	1.7185 +/- 0.0022 ns
7	10.1982 ns	10.1969 +/- 0.0158 ns
8	6.1221 µs	6.1115 +/- 0.0073 µs
9	10.1715 ns	10.1714 +/- 0.0138 ns
10	9.6725 ns	9.6752 +/- 0.0115 ns

Experimental Rossi-α

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Experiment	Measured α (ms^{-1})	ACODE α (ms^{-1})	Progenitor α (ms^{-1})
Godiva	-1110 +/- 20	-1030 +/- 60	-1136 +/- 12
Jezebel	-640 +/- 10	-510 +/- 120	-643 +/- 13
Flattop-23	-267 +/- 5	-252 +/- 30	-296 +/- 5
BIG TEN	-117 +/- 1	-120 +/- 5	-122 +/- 2.5
STACY-29	-0.122 +/- 0.004	--	-0.128 +/- 0.002
WINCO-5	-1.1093 +/- 0.0003	--	-1.153 +/- 0.037

ENDF/B-VI.5 nuclear data

Linear Perturbation

Perturbation Theory

- Suppose we want to know the change in reactivity due to a small change in a system
  - Density changes
  - Enrichment/concentration uncertainties
  - Cross section data changes

- Can use linear perturbation theory

$$\Delta\rho = -\frac{\langle \psi^{\dagger} \mathbf{P} \psi \rangle}{\langle \psi^{\dagger} \mathbf{F} \psi \rangle}$$

$$\mathbf{P} = \Delta\Sigma_t - \Delta\mathbf{S} - \frac{1}{k}\Delta\mathbf{F}$$

Limitations in MCNP Perturbation

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- **MCNP perturbations do not account for fission source perturbations.**
  - See talk by Jeff Favorite
- **Linear perturbation theory can account for this**
- **Approach still has some limitations in this respect**
  - Large local flux shifts cause problems

Reactor Kinetics Tallies

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- **Adjoint-weighted collision rate perturbation:**

$$\left\langle \psi^\dagger \Delta\Sigma_t \psi \right\rangle = \frac{1}{W} \sum_p \pi_p \sum_{\tau \in p} \Sigma_{t,\tau} w_{0,p} d_\tau$$

- **Adjoint-weighted scattering source perturbation:**

$$\left\langle \psi^\dagger \Delta \mathbf{S} \psi \right\rangle = \frac{1}{W} \sum_p \pi_p \sum_{s \in p} \frac{\Delta\Sigma_s}{\Sigma_s} w_{0,p}$$

- **Adjoint-weighted fission source perturbation:**

$$\left\langle \psi^\dagger \frac{1}{k} \Delta \mathbf{F} \psi \right\rangle = \frac{1}{W} \sum_p \pi_p \sum_{f \in p} \frac{\Delta \nu\Sigma_f}{\nu\Sigma_f} w_{0,p}$$

Perturbation Validation

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- Boron-10 worth in 2D APWR quarter core.
- ^{10}B concentration: 1.675 x 10^{-4} to 1.65 x 10^{-4}

k_{eff}	0.99983 +/- 0.00008
Ref. Δk	0.00325 +/- 0.00011
Calc. Δk	0.00320 +/- 0.00011

Ref. obtained by comparing k_{eff} for two separate MCNP runs.

Perturbation Validation

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- Godiva ^{235}U sphere
- Change cross section library from ENDF/B-VI.5 to ENDF/B-VII.0

k_{eff}	0.99646 +/- 0.00004
Ref. Δk	0.00344 +/- 0.00006
Calc. Δk	0.00358 +/- 0.00006

Ref. obtained by comparing k_{eff} for two separate MCNP runs.

Perturbation Validation

- Unit fuel cell with control rod
- Change enrichment by +0.01%

k_{eff}	1.00390 +/- 0.00017
Ref. Δ*k*	0.00125 +/- 0.00025
Calc. Δ*k*	0.00131 +/- 0.00025

Ref. obtained by comparing k_{eff} for two separate MCNP runs.

Perturbation Validation

- Unit fuel cell with control rod
- No ^{131}Xe to 5 ppb ^{131}Xe in fuel

k_{eff}	1.00368 +/- 0.00014
Ref. Δ*k*	-0.01687 +/- 0.00020
Calc. Δ*k*	-0.01722 +/- 0.00020

Ref. obtained by comparing k_{eff} for two separate MCNP runs.

Summary & Future Work

Summary

- **Adjoint weighting is useful for calculating critical system parameters**
- **New method extends Monte Carlo to do adjoint-weighted tallies**
- **Applied to:**
  - Kinetics parameters
  - Linear perturbations of reactivity

Future Work

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- Further application to sensitivity/uncertainty analysis
- Subcritical source importance weighting

Questions?

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