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<i>Title:</i>	A Tutorial on Using MCNP for 1-Group Transport Calculations
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# A Tutorial on Using MCNP for 1-group Transport Calculations

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## A Tutorial on Using MCNP for 1-Group Transport Calculations

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1-group transport calculations are an invaluable R&D tool for developing new algorithms and calculational methods for particle transport applications. They are also important for verifying the correctness of existing transport codes, since many 1-group transport analytical problems exist for which exact solutions are known. MCNP is a general-purpose, 3D, continuous-energy Monte Carlo particle transport code used for a wide variety of applications. In this tutorial, we describe how MCNP can be used to perform 1-group transport calculations. An auxiliary code called ONEGXS is first used to construct 1-group cross-section files in the ACE format required by MCNP. Then, MCNP can be run in a multigroup mode using the 1-group data files. This procedure is illustrated by several examples. It is important for users to understand the limitations on the MCNP Monte Carlo methods: For 1-group problems with isotropic scattering, MCNP results match exact analytical solutions. For linearly anisotropic scattering, MCNP results match exact analytical solutions when  $|\bar{\mu}| \leq \frac{1}{3}$ , but will generally not match exact analytical solutions otherwise, due to the non-physical effects of truncating the scattering data representation.

- **1-Group Transport Calculations - Background**
  - Importance
  - References
- **Generating 1-Group Cross-section Files**
  - onegxs code
  - Input & output
  - Examples
- **Running 1-Group Calculations with MCNP**
  - Multigroup calculations
  - Referencing 1-group datafiles
  - Examples
- **Discussion**
  - Angular scattering distributions
  - $P_0$  and  $P_1$  scattering
  - Truncation errors for highly forward- or backward-peaked scatter

- **1-group transport calculations are an important R&D tool**
  - Useful for developing new algorithms & computational methods that do not involve energy-dependence
    - Examples:            geometry tracking, criticality, void streaming
  - Simplifies & speeds up calculations
- **1-group calculations are important for verifying correctness of codes**
  - Many transport problems have known, exact, analytical solutions
  - Typically, these are 1-group with simple geometry

- **Criticality benchmarks**

- A. Sood, R.A. Forster, D.K. Parsons, "Analytic Benchmark Test Set for Criticality Code Verification", LA-UR-01-3082 (2001)
- Compilation of 75 criticality problems with exact analytical solutions
  - **30 problems - 1-group, isotropic scatter**
  - **13 problems - 1-group, anisotropic scatter**
  - 26 problems - 2-group, isotropic scatter
  - 4 problems - 2-group, anisotropic scatter
  - 1 problem - 3-group, isotropic scatter
  - 1 problem - 6-group, isotropic scatter

- **Kobiyashi void-streaming benchmark**

- K. Kobayashi, N. Sugimura, Y. Nagaya, "3-D Radiation Transport Benchmark Problems and Results for Simple Geometries with Void Regions", OECD/NEA (2000).

- **Many others .....**

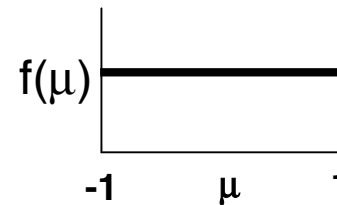
- **MCNP reads ACE-format cross-section files to obtain cross-section data, scattering distributions, etc., required for performing transport calculations**
  - ACE files - described in MCNP manual, Volume III, Appendix F
  - Very general, to accommodate continuous-energy neutron/photon/electron data & also multigroup data
  - Compact (but complex) data layout
- **onegxs**
  - Auxilliary code to simplify preparing 1-group cross-section datafiles
    - Includes basic cross-section data:  
total, scatter, capture, fission, nu
    - Provides scattering distribution data in the lab system:  
isotropic ( $P_0$ ) or linearly-anisotropic ( $P_1$ ) scattering
  - Creates datafile in ACE format

# Generating 1-Group Cross-section Files

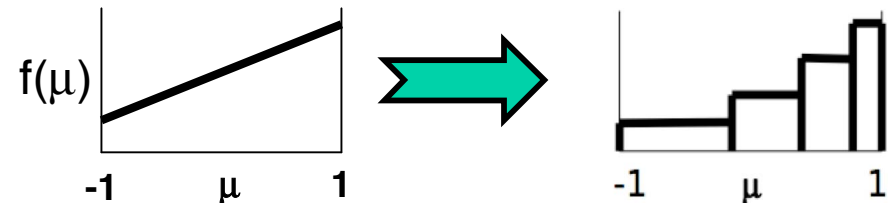
- Scattering distributions from onegxs are either isotropic ( $P_0$ ) or linearly anisotropic ( $P_1$ ) in the lab system for multigroup MCNP

- Isotropic ( $P_0$ )**

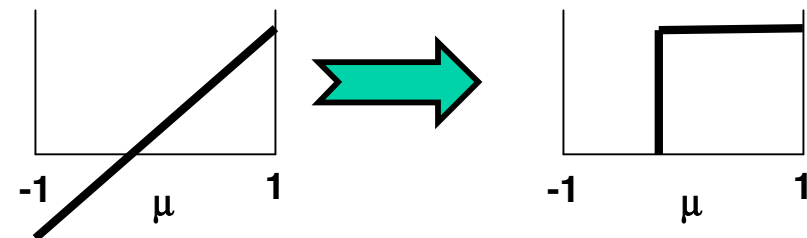
- Flag, no data needed
- Exact form used in MCNP



- $P_1$  scatter, with  $|\bar{\mu}| \leq 1/3$** 
  - 1000 steps, equal-area



- $P_1$  scatter, with  $|\bar{\mu}| > 1/3$** 
  - Linear PDF replaced by 1 step
  - Step bounds chosen to preserve  $\bar{\mu}$  & remain positive





- **OneGXS**

- Takes user-specified, one-group cross sections & writes a cross section file that MCNP can read.
- The program is driven from the command prompt & has a variety of input options. Use "**onegxs -h**" for help.
- It can handle multiplying or non-multiplying media, and isotropic or linearly anisotropic scattering.

- **Instructions for Use**

- OneGXS accepts arguments through a flag, followed by data.
- The one exception is that the first argument is the name of the output file and does not use a flag.
- General form:

```
onegxs   filename [-z suffix]           \  
          [-t sigt] [-c sigc] [-s sigs]    \  
          [-n nu  -f sigf]                 \  
          [-s0 sigs] [-s1 sigs1] [-u mubar]
```

- File** Name of the output file that MCNP will read & always required. Should be the first item listed.
- c** This flag indicates that the next item in the command line is the capture cross section. The capture cross section is defined as the absorption minus the fission,  $\Sigma_C = \Sigma_A - \Sigma_F$
  - f** This flag indicates that the next item in the command line is the fission cross section,  $\Sigma_F$
  - h** This flag will print the help page, with a list of all options available for running the program. After displaying the help page, the program will immediately exit. Note: the help page will also be displayed if no arguments are given.
  - n** This flag indicates that the next item in the command line is  $\nu$  (nu-bar), the average number of neutrons produced in each fission event.
  - s** This flag indicates that the next item in the command line is the scattering cross section,  $\Sigma_S$ . This is the isotropic, or  $P_0$ , scattering cross section.
  - s0** This flag indicates that the next item in the command line is the  $P_0$ , or isotropic, scattering cross section,  $\Sigma_{S0}$ . This is the same value as the previous option.
  - s1** This flag indicates that the next item in the command line is the  $P_1$  scattering cross section,  $\Sigma_{S1}$ . This provides the next term in the Legendre expansion, causing the scattering to be anisotropic.
  - t** This flag indicates that the next item in the command line is the total cross section. The total cross section is defined as the sum of the capture, fission, and  $P_0$  scattering cross sections,  
$$\Sigma_T = \Sigma_A + \Sigma_S = \Sigma_C + \Sigma_F + \Sigma_S$$
  - u** This flag indicates that the next item in the command line is  $\mu$  (mu-bar), the average cosine of the scattering angle,  $\mu = \Sigma_{S1} / \Sigma_{S0} = \Sigma_{S1} / \Sigma_S$ . If mu-bar is zero, scattering is isotropic. Note: special caution should be taken with mu-bar, since problems with  $|\mu\text{-bar}| > 1/3$  can give wrong answers. Please read the section on mu-bar.
  - z** This flag indicates that the next item in the command line is the two digit material identifier that accompanies the ZAIID number in MCNP. The ZAIID will always be 92250. (e.g. 92250.32m)

- **Using onegxs**
  - Not all of the options are needed to run the program.
  - If the program can not calculate what it needs from the given data, then it will exit.
  - Redundant data is checked against the overlapping data for differences.
- **Onegxs screen output, example**

**Created MCNP cross section file:**

**xsec02**

**The following cross sections were used:**

**sigT= 0.3264, sigF= 0.0816, sigC= 0.019584, sigS0= 0.225216, nu= 3.24, uBar= 0**

**Please use the following line in your MCNP deck on the xsn card:**

**92250.02m 1.0 xsec02 0 1 1 11 0 0 0.0**

# Running 1-Group Calculations with MCNP

- Run MCNP in multigroup mode

- Don't use the MODE card (assume neutron multigroup)

- Importances should all be given in the form: **imp:n**

- Use default energy for any SDEF source cards

- Supply the MGOPT card as one of the data cards

```
mgopt f 1
```

- Supply XSn cards in the MCNP input to describe the xsec files

```
xs1 92250.12m 1.0 xsfile1 0 1 1 13 0 0 0.0  
xs2 92250.27m 1.0 xsfile2 0 1 1 1013 0 0 0.0  
xs3 92250.01m 1.0 xsfile3 0 1 1 11 0 0 0.0
```

Values in **red** must match the output information from **onegxs**

# Example - MCNP Input

## PUa-1-1-SL Criticality Benchmark Problem

PUa-1-1-SL

10 1 1.0 -1 2 \$SEMI-INFINITE SLAB

20 0 1:-2 \$OUTSIDE WORLD

1 px 0.77032

2 px -0.77032

kcode 20000 1.0 50 550

ksrc 0.0 0.0 0.0

hsrc 1 -1e20 1e20 1 -1e20 1e20 1 -1e20 1e20

xs1 92250.32m 1.0 xsec32 0 1 1 1013 0 0 0.0

m1 92250.32m 1.0

imp:n 1 0

mgopt f 1

**Problem PUa-1-0-SL  
from  
Analytic Criticality  
Benchmark Set**

# Example 1

- **Problem 2 from the analytic criticality benchmark set**

- Identifier: **PUa-1-0-SL**

- 1-group slab, with **isotropic scatter (in lab)**

- From Table 2 in reference:

$$\Sigma_T = .326400, \quad \Sigma_S = .225216, \quad \Sigma_C = .019584,$$

$$\Sigma_F = .081600, \quad \nu = 3.24$$

- From Table 3 in reference:

$$\text{critical half-thickness} = 1.853722$$

- Exact eigenvalue:  $K_{\text{eff}} = 1.000000$

# Example 1

- **onegxs input for problem PUa-1-0-SL**

```
onegxs xsec02 -z 02      -t .326400    -s .225216  \  
                        -c .019584    -f .081600  -n 3.24
```

- **onegxs screen output for problem PUa-1-0-SL**

Created MCNP cross section file:

```
xsec02
```

The following cross sections were used:

```
sigT= 0.3264, sigF= 0.0816, sigC= 0.019584, sigS0= 0.225216, nu= 3.24, uBar= 0
```

Please use the following line in your MCNP deck on the xsn card:

```
92250.02m 1.0 xsec02 0 1 1 11 0 0 0.0
```



# Example 1

- onegxs file created for problem PUA-1-0-SL (xsec02)

```
92250.02m      250      0  20070629
sigT= 0.3264, sigF= 0.0816, sigC= 0.019584, sigS0= 0.225216, nu= 3.24, uBar= 0
```

```
  11    92250      0      0      1      0      0      0
   0      1      0      1      0      0      0      0
   1      3      4      5      6      7      0      0
   0      0      0      0      8      0      0     10
  11      0      0      0      0      0      0      0
   0      0      0      0      0      0      0      0
5.000000000000e+01  1.000000000000e+02  3.264000000000e-01  8.160000000000e-02
3.240000000000e+00  1.000000000000e+00  1.958400000000e-02  9.000000000000e+00
2.252160000000e-01  0.000000000000e+00  0.000000000000e+00
```

**For isotropic scattering, no angular distribution data is needed**

# Example 1

- MCNP input file for problem PUa-1-0-SL, for 500M histories

**PUa-1-0-SL**

**10 1 1.0 -1 2 \$SEMI-INFINITE SLAB**

**20 0 1:-2 \$OUTSIDE WORLD**

**1 px 1.853722**

**2 px -1.853722**

**kcode 20000 1.0 50 2550**

**hsrc 5 -1.853722 1.853722 1 -1e20 1e20 1 -1e20 1e20**

**ksrc 0.0 0.0 0.0**

**xs1 92250.02m 1.0 xsec02 0 1 1 11 0 0 0.0**

**m1 92250.02m 1.0**

**imp:n 1 0**

**mgopt f 1**

# Example 1

- MCNP output for problem PUA-1-0-SL

```
cycle      k(col)      ctm  entropy  active      k(col)      std dev      fom
   1      1.11109      0.01  2.20E+00
   2      1.01883      0.03  2.29E+00
   3      1.02007      0.04  2.31E+00
. . . . .
2544      1.00112      38.41  2.31E+00      2494      1.00004      0.00012      1707942
2545      0.98849      38.43  2.31E+00      2495      1.00003      0.00012      1706282
2546      0.99955      38.44  2.31E+00      2496      1.00003      0.00012      1706999
2547      0.99067      38.46  2.31E+00      2497      1.00003      0.00012      1706142
2548      0.99723      38.47  2.30E+00      2498      1.00003      0.00012      1706712
2549      1.00831      38.49  2.31E+00      2499      1.00003      0.00012      1706216
2550      0.99519      38.50  2.31E+00      2500      1.00003      0.00012      1706499
. . . . .
final k(col/abs/trk len) = 1.00009      std dev = 0.00010
```

**For 1-group problems with isotropic scattering, results using onegxs + MCNP match the exact analytic solution (within statistics)**

**Problem Ua-1-1-CY  
from  
Analytic Criticality  
Benchmark Set**

## Example 2

- **Problem 36 from the analytic criticality benchmark set**

- Identifier: **Ua-1-1-CY**

- 1-group infinite cylinder, with **anisotropic scatter (in lab)**,  $\bar{\mu} = .171052631$

- From Table 23 in reference:

$$\Sigma_T = .32640, \quad \Sigma_S = .248064, \quad \Sigma_{S1} = .042432$$

$$\Sigma_C = .013056, \quad \Sigma_F = .065280, \quad \nu = 2.70$$

- From Table 24 in reference:

$$\text{critical radius} = 5.514296811$$

- Exact eigenvalue:  $K_{eff} = 1.000000$

## Example 2

- **onegxs input for problem Ua-1-1-CY**

```
onegxs xsec36 -z 36      -t .326400      -s .248064  \  
                        -c .013056      -f .065280  -n 2.70  \  
                        -s1 .042432
```

- **onegxs screen output for problem Ua-1-1-CY**

Created MCNP cross section file:

```
xsec36
```

The following cross sections were used:

```
sigT= 0.3264, sigF= 0.06528, sigC= 0.013056, sigS0= 0.248064, nu= 2.7, uBar= 0.171053
```

Please use the following line in your MCNP deck on the xsn card:

```
92250.36m 1.0 xsec36 0 1 1 1013 0 0 0.0
```

## Example 2

- onegxs file created for problem Ua-1-1-CY (xsec36)

```
92250.36m          250          0  20070702
sigT= 0.3264, sigF= 0.06528, sigC= 0.013056, sigS0= 0.248064, nu= 2.7, uBar= 0.171053
```

```

1013  92250  1001  0  1  0  0  0
  0  1  0  1  0  0  0  0
  1  3  4  5  6  7  0  0
  0  0  0  0  8  0  0  10
 12  0  0  0  0  0  0  0
  0  0  0  0  0  0  0  0
5.000000000000e+01  1.000000000000e+02  3.264000000000e-01  6.528000000000e-02
2.700000000000e+00  1.000000000000e+00  1.305600000000e-02  9.000000000000e+00
2.480640000000e-01  1.100000000000e+01  1.000000000000e+00  1.300000000000e+01
-1.000000000000e+00 -9.959007501602e-01 -9.918190625466e-01 -9.877547078747e-01
-9.837074733325e-01 -9.796771402444e-01 -9.756634988818e-01 -9.716663483741e-01
. . . . .
9.894070569220e-01  9.907332609147e-01  9.920588675273e-01  9.933838764883e-01
9.947082892934e-01  9.960321076894e-01  9.973553312983e-01  9.986779607410e-01
1.000000000000e+00
```

For linearly-anisotropic scatter, with  $|\mu\text{-bar}| < 1/3$ ,  
1000 equiprobable steps are used to represent the angular distribution

## Example 2

- MCNP input file for problem Ua-1-1-CY, for 500M histories

Ua-1-1-CY

1 1 1.0 -1 \$SEMI-INFINITE CYLINDER

2 0 1 \$OUTSIDE WORLD

1 rcc 0 0 0 0 0 1e12 5.514296811

imp:n 1 0

kcode 20000 1.0 50 2550

hsrc 1 -1e20 1e20 1 -1e20 1e20 1 -1e20 1e20

ksrc 0.0 0.0 .5e12

xs1 92250.36m 1.0 xsec36 0 1 1 1013 0 0 0.0

m1 92250.36m 1.0

mgopt f 1



## Example 2

- MCNP output for problem Ua-1-1-CY

```
cycle      k(col)      ctm  entropy  active      k(col)      std dev      fom
   1      1.23829      0.01  0.00E+00      2494      0.99995      0.00010      2066295
   2      1.09388      0.02  0.00E+00      2495      0.99995      0.00010      2067075
   3      1.03832      0.02  0.00E+00      2496      0.99995      0.00010      2062917
. . . . .
2544      0.99772      45.99  0.00E+00      2497      0.99995      0.00010      2062640
2545      1.00139      46.01  0.00E+00      2498      0.99995      0.00010      2062971
2546      1.01269      46.03  0.00E+00      2499      0.99995      0.00010      2063786
2547      0.99399      46.05  0.00E+00      2500      0.99995      0.00010      2064459
2548      0.99587      46.07  0.00E+00
2549      1.00074      46.08  0.00E+00
2550      0.99770      46.10  0.00E+00
. . . . .
final k(col/abs/trk len) = 1.00005      std dev = 0.00008
```

**For 1-group problems with linearly-anisotropic scattering, with  $|\mu\text{-bar}| < 1/3$ , results using onegxs + MCNP match the exact analytic solution (within statistics)**

**Problem Ub-1-1-CY  
from  
Analytic Criticality  
Benchmark Set**

## Example 3

- **Problem 37 from the analytic criticality benchmark set**

- Identifier: **Ub-1-1-CY**

- 1-group infinite cylinder, with **anisotropic scatter (in lab)**,  $\bar{\mu} = .855263157$

- From Table 23 in reference:

$$\Sigma_T = .32640, \quad \Sigma_S = .248064, \quad \Sigma_{S1} = .212160$$

$$\Sigma_C = .013056, \quad \Sigma_F = .065280, \quad \nu = 2.70$$

- From Table 24 in reference:

$$\text{critical radius} = 6.940205668$$

- Exact eigenvalue:  $K_{eff} = 1.000000$

## Example 3

- **onegxs input for problem Ub-1-1-CY**

```
onegxs xsec37 -z 37      -t .326400    -s .248064  \  
                        -c .013056    -f .065280  -n 2.70  \  
                        -s1 .212160
```

- **onegxs screen output for problem Ub-1-1-CY**

Created MCNP cross section file:

```
xsec37
```

The following cross sections were used:

```
sigT= 0.3264, sigF= 0.06528, sigC= 0.013056, sigS0= 0.248064, nu= 2.7, uBar= 0.855263
```

Please use the following line in your MCNP deck on the xsn card:

```
92250.37m 1.0 xsec37 0 1 1 14 0 0 0.0
```

## Example 3

- onegxs file created for problem Ub-1-1-CY (xsec37)

```
92250.37m          250          0  20070702
sigT= 0.3264, sigF= 0.06528, sigC= 0.013056, sigS0= 0.248064, nu= 2.7, uBar= 0.855263
```

```
14  92250  2  0  1  0  0  0
 0  1  0  1  0  0  0  0
 1  3  4  5  6  7  0  0
 0  0  0  0  8  0  0  10
12  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0
5.000000000000e+01  1.000000000000e+02  3.264000000000e-01  6.528000000000e-02
2.700000000000e+00  1.000000000000e+00  1.305600000000e-02  9.000000000000e+00
2.480640000000e-01  1.100000000000e+01  1.000000000000e+00  1.300000000000e+01
7.105263157895e-01  1.000000000000e+00
```

For linearly-anisotropic scatter, with  $|\mu\text{-bar}| > 1/3$ , the angular distribution is represented by 1 step is used, with bounds chosen to preserve  $|\mu\text{-bar}|$

## Example 3

- MCNP input file for problem PUa-1-0-SL, for 500M histories

```
Ub-1-1-CY
```

```
1 1 1.0 -1 $SEMI-INFINITE CYLINDER
2 0 1 $OUTSIDE WORLD
```

```
1 rcc 0 0 0 0 0 1e12 6.940205668
```

```
imp:n 1 0
```

```
kcode 20000 1.0 50 2550
```

```
hsrc 1 -1e20 1e20 1 -1e20 1e20 1 -1e20 1e20
```

```
ksrc 0.0 0.0 .5e12
```

```
xs1 92250.37m 1.0 xsec37 0 1 1 14 0 0 0.0
```

```
m1 92250.37m 1.0
```

```
mgopt f 1
```

# Example 3

- MCNP output for problem Ub-1-1-CY

```
cycle      k(col)      ctm  entropy  active      k(col)      std dev      fom
   1      1.22206      0.02  0.00E+00
   2      1.09928      0.04  0.00E+00
   3      1.05035      0.06  0.00E+00
. . . . .
2544      1.01522      45.23  0.00E+00      2494      1.01865      0.00010      2560777
2545      1.01316      45.25  0.00E+00      2495      1.01865      0.00010      2560448
2546      1.01781      45.27  0.00E+00      2496      1.01865      0.00010      2561446
2547      1.01126      45.28  0.00E+00      2497      1.01864      0.00010      2560008
2548      1.01934      45.30  0.00E+00      2498      1.01864      0.00010      2561015
2549      1.01609      45.32  0.00E+00      2499      1.01864      0.00010      2561737
2550      1.01955      45.34  0.00E+00      2500      1.01864      0.00010      2562728
. . . . .
final k(col/abs/trk len) = 1.01870      std dev = 0.00008
```

**For 1-group problems with linearly-anisotropic scattering, with  $|\mu\text{-bar}| > 1/3$ , results using onegxs + MCNP will generally NOT match the exact analytic solution**

# Angular Scattering Distributions & Truncation Errors



- **Scattering angle PDFs**

- Scattering distributions are usually represented by a Legendre polynomial expansion in data libraries (eg, ENDF/B)

$$f(\mu) = \sum_{n=0}^N \frac{2n+1}{2} a_n P_n(\mu), \quad -1 \leq \mu \leq 1$$

- Typically N=20 Legendre moments  $a_n$  are stored
- The Legendre representation is converted to histogram or piecewise-linear PDFs for use in Monte Carlo codes by NJOY or some other processing program

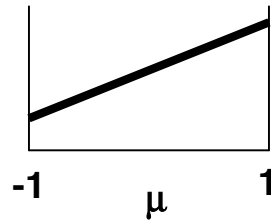
- **Sometimes, truncated angular PDFs must be used**

- $S_N$  codes typically use P1, P3, P5, ..., truncated expansions
- May want to compare MC &  $S_N$  using same cross-section data
- Truncated Legendre expansions may produce angular PDFs with negative data over part of range

- Simplest case - P1 scattering

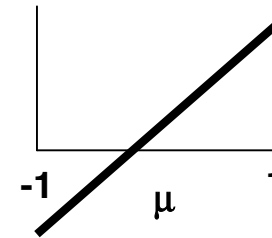
$$f(\mu) = \frac{1 + a\mu}{2}, \quad -1 \leq \mu \leq 1, \quad a = 3\bar{\mu}$$

$$|\bar{\mu}| \leq \frac{1}{3}$$



$f(\mu) > 0$  in  $[-1, 1]$

$$|\bar{\mu}| > \frac{1}{3}$$



$f(\mu) < 0$  for some  $\mu$

- If  $|\bar{\mu}| > \frac{1}{3}$ , need to replace  $f(\mu)$  by another PDF  $g(\mu)$

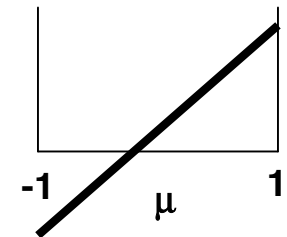
- Non-negative PDF:

$$g(\mu) \geq 0$$

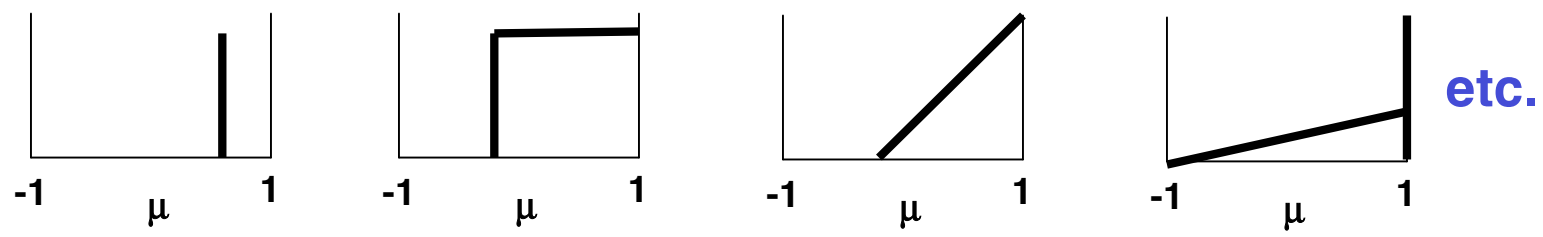
- Preserve physical condition:

$$\int_{-1}^1 \mu g(\mu) d\mu = \bar{\mu}$$

- Non-physical PDF, linear with  $|\bar{\mu}| > \frac{1}{3}$



- Alternate forms of PDF that can preserve 1st moment



- All of these can be chosen to preserve the physical data,  $\langle \mu \rangle$
- The second form, a 1-step histogram, can be accommodated by MCNP using the standard data formats, and does not require code modifications

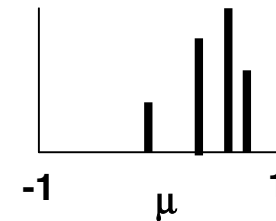
For  $\bar{\mu} > \frac{1}{3}$ , choose bounds  $[ 2\bar{\mu} - 1, 1 ]$

For  $\bar{\mu} < -\frac{1}{3}$ , choose bounds  $[ -1, 2\bar{\mu} + 1 ]$

- For higher order truncated Legendre expansions, more elaborate schemes must be used to preserve the moments & guarantee non-negative PDFs

- For a  $P_N$  expansion, can find  $(N+1)/2$  discrete angles which preserve all  $N$  moments & have positive weights

- Use generalized orthogonal polynomials
- See description in MORSE code manual



- Can use nonlinear least-squares fitting to find equiprobable histogram distribution that approximately preserves moments

