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On the Definition of Neutron Lifetimes in Multiplying and Nonmultiplying Systems

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Abstract – *In this work, we define the various time constants (i.e., life spans, lifetimes, and generation times) associated with the life of a neutron in non-multiplying and multiplying systems.*

I. INTRODUCTION

For the last 50 or so years, the nuclear industry has been trying to validate nuclear cross section sets using k_{eff} measurements obtained from a wide variety of benchmark critical-mass experiments. However, benchmarking against k_{eff} is not as beneficial as one might hope since k_{eff} is merely the ratio of the neutron production rate to the neutron loss rate. As such, the absolute value of the cross sections in any given library may be significantly in error and still yield reasonably accurate estimates of k_{eff} if the ratio of $\nu\Sigma_f/\Sigma_a$ is correct. Although a k_{eff} measurement is a necessary and vital measurement, by itself, it is not sufficient to validate any cross section set; a favorable comparison between a measured and calculated k_{eff} only tests one aspect of that cross section set—namely, the ratio of $\nu\Sigma_f/\Sigma_a$. To validate other aspects of a cross section set, other integral quantities such as neutron lifetime, effective delayed neutron fraction, fission ratios, central-worth reactivity worths, neutron spectra, etc. must also be measured.

A neutron lifetime measurement is particularly useful in this regard because the neutron lifetime in both multiplying and nonmultiplying systems varies approximately as $1/\Sigma_a$ and can be easily and accurately measured using standard die-away techniques. However, some care must be exercised when comparing measured and calculated lifetimes for the following two reasons.

First, the proper interpretation of a neutron lifetime measurement in a complex system is quite complicated and, judging from the literature, often misconstrued. For example, in some types of reflected systems, spatial effects are prevalent and several decay constants may be observed. Often, these measurements are analyzed using *one-region* expressions which, in most cases, do not apply. It is not surprising that comparison between the measured and calculated life-

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times for these types of systems indicates poor agreement.

And second, one can formulate numerous lifetimes corresponding to various reactions occurring within a given system and can calculate these various lifetimes either as unweighted quantities or as adjoint-weighted quantities. For small, bare systems, the difference between an unweighted and an adjoint-weighted lifetime can be relatively small—5 to 10%—whereas, in large, complex systems, the difference can be several orders of magnitude. These differences may be further exacerbated depending on how the neutron and adjoint fluxes used in the evaluations have been determined; the eigenfunctions obtained from an α-eigenvalue solution and a k-eigenvalue solution can be significantly different for systems in which $k>>1$. These differences in the eigenfunctions can lead to differences of several orders of magnitude in a lifetime calculation

Thus, not only is it important to understand which lifetime is measured during an experiment, but it is just as important to understand how to calculate a specific lifetime using Monte Carlo and/or deterministic transport solutions; after all, a comparison between a measured and a calculated lifetime can only be meaningful it the two quantities are the same.

Historical Background and Previous Works

It is unknown who first defined a neutron lifetime in a quantitative sense. However, estimates of the neutron lifetime in a thermal system and in a fast system are mentioned in the Frisch-Peierls Memorandum[1] written in early 1940. By the time the Los Alamos National Laboratory was formed in March 1943, the basic concept that the neutron lifetime is inversely proportional to the absorption rate must have been well understood as evidenced by the mundane manner in which Serber[2] discussed it in the Los Alamos Primer, LA-1.[b] Serber defined the neutron lifetime as the 'mean time between fissions' based on two assumptions: 1) the system was so large that neutrons would not be lost by leakage and 2) all neutrons absorbed in the system would only be absorbed in fission reactions (i.e., no parasitic absorption). Hence, Serber's lifetime definition corresponded to

$$\tau = \frac{1}{v\Sigma_f} \quad , \tag{1}$$

where v is the average neutron velocity and Σ_f is the average macroscopic fission cross section.

In October 1943, Feynman[3] extended Serber's definition to an *ideal* 1/v absorber, including both parasitic absorption and fission absorption. Feynman defined the absorption lifetime in the same manner as Serber, but used the macroscopic absorption cross section rather than the fission cross section. He also stated that the reciprocal of the absorption lifetime was, by definition, the 'probability per second of being captured.' In that same report, Feynman formulated the the-

[b.] The Los Alamos Primer, LA-1, is a collection of lecture notes used by Serber to indoctrinate new recruits into the Manhattan project in April, 1943. It is now an unclassified document readily available to the public.

ory for the 1/v–absorption technique that is still occasionally used to measure the absorption lifetime in a multiplying system.

About a year later, de Hoffmann et al.[4] developed a more general expression for the definition of a fission lifetime. In their derivation, de Hoffmann et al. defined P_p as the probability of a prompt neutron, emitted at time t', causing a fission at any later time. The chance of producing a fission at time t was found by weighting P_p by a function $R(t - t')dt$ such that $P_pR(t - t')dt$ is the probability that a neutron born promptly at time t' will produce a fission in the interval dt at time t. By writing down an expression governing the variation of the time-dependent fission rate and expanding this expression in a Taylor series, de Hoffmann et al. showed that the 'average time from one fission to the next fission due to prompt neutrons' is given by

$$\bar{T}_f = \int_0^\infty (t-t')R(t-t')dt \quad , \tag{2}$$

where the integral of $R(t - t')dt$ is appropriately normalized to 1.0.

In March 1946, Kupferberg[5] wrote another Los Alamos report in which the neutron lifetime for a nonmultiplying reflector was defined such that the loss rate due to absorption and leakage were both included. In his report, Kupferberg stated (without any formal derivation) that the *mean* neutron-lifetime in the reflector, τ_r, is

$$\frac{1}{\tau_r} = \frac{1}{\tau_a} + \frac{1}{\tau_l} \quad , \tag{3}$$

where $1/\tau_a$ is the probability per unit time of an absorption and $1/\tau_l$ is the probability per unit time of a leakage. (Note, we have changed Kupferberg's original nomenclature to maintain consistency within this manuscript.) In accordance with Kupferberg's definition, $1/\tau_r$ is the probability per unit time of either an absorption or a leakage.

During the period of time from 1943 to 1947, several techniques were developed to measure the neutron lifetime in multiplying and nonmultiplying systems. These included the 1/v-absorber technique, the rod-oscillator technique, the pulsed-neutron technique, and the Rossi-α technique developed by Feynman[6] in July, 1946. In his derivation of the Rossi-α equation, Feynman used the fission lifetime, τ_f, as the basic measure of the time duration of a prompt fission chain. In January 1947, Baker et al.[7] rewrote Feynman's Rossi-α equation in terms of the prompt multiplication factor and the 'mean life of a neutron', which they defined as $\tau = \tau_f / \bar{\nu}_p$ where $\bar{\nu}_p$ is the average number of prompt neutrons released per fission.

In 1948, Hurwitz[8] presented his derivation of the point kinetic equations in which he defines a quantity τ that he refers to as the 'prompt neutron generation time.' Later on in the manuscript, Hurwitz describes τ as the 'average time it takes a prompt neutron to produce a fis-

sion.' He also derives an expression to calculate τ which is very similar in form to the expression derived by de Hoffmann et al.[4]

$$\tau = \frac{\int TP(\boldsymbol{r}, \boldsymbol{r}', T)dTdV}{\int P(\boldsymbol{r}, \boldsymbol{r}', T)dTdV} \quad , \tag{4}$$

where $P(\boldsymbol{r},\boldsymbol{r}',T)$ is the probability that a neutron liberated in a fission at point $\boldsymbol{r}'$ and time $t = 0$ gives rise to a fission at point $\boldsymbol{r}$ and time t, per unit time and volume, T is the time from birth-to-fission and V is the system volume.

During the 1950s there was an *explosion* of articles written on the subject of reactor physics and reactor kinetics in which some type of neutron lifetime was invariably defined. The definitions were usually based on diffusion theory, or on the somewhat nebulous *life-cycle* model in which neutrons in one *generation* (which is not well defined) produced neutrons in the next *generation*. Several textbooks on reactor theory began to appear in the early 1950s which propagated the life-cycle model of neutron reproduction and repeated many of the same definitions previously mentioned in this section. The most common expression for the neutron lifetime found in the literature during this era was the expression derived from one-group diffusion theory.

$$\tau = \frac{1}{v\Sigma_a(1 + L^2B^2)} \quad . \tag{5}$$

In some cases, slightly different slants were incorporated into the verbal definition of a neutron lifetime and/or a neutron generation time. For example, Glasstone and Edlund[9] stated that a neutron generation time is "the average time between successive neutron generations and is equal to the sum of the slowing down time of the fast fission neutrons and of the diffusion time, or lifetime, of the thermal neutrons." From this definition, we infer that the lifetime must be a subset of the neutron generation time. One paragraph later, Glasstone and Edlund further stated that "for a reactor in which a considerable proportion of the fissions are caused by neutrons with energies above thermal, the generation time is defined as the average time between successive fissions of all types." Based on this statement, one is drawn to another conclusion whereby the generation time must be identically equal to the fission lifetime as defined by Serber.[2]

In 1952, Weinberg[10] introduced the concept of using an adjoint flux when solving the neutron transport equation. In that work, Weinberg talked about the λ eigenvalue (which we believe is equivalent to $k_{eff} - 1$) and briefly discussed the time-dependent solution. He qualitatively

defined an adjoint-weighted neutron lifetime to be "the time required for a neutron to go through a complete multiplication cycle or generation."

In 1955, Ussachoff[11] derived the point-kinetic equation from the Boltzmann transport equation. From that derivation, he obtained the following expression for the neutron generation time, Λ, written in terms of the angular fluxes and angular adjoint fluxes.

$$\Lambda = \frac{\int \frac{\Phi^{+}\Psi}{v}\, d\Omega dV dE}{\int \Phi^{+}\chi_f \bar{\nu}_t \Sigma'_f \Psi'\, d\Omega dE' dV dE} \quad . \tag{6}$$

(As before, we have changed the original nomenclature to be consistent with the nomenclature in this manuscript.) Ussachoff described this quantity as "the mean lifetime of the neutron in the reactor."

In 1958, Weinberg and Wigner[12] published their book on reactor theory in which they dedicated an above–average number of pages to the discussion of neutron lifetimes and neutron generation times. They quickly pointed out that it is very difficult to rigorously define a neutron lifetime based solely on the concepts of the life–cycle model since a *generation* time is not well defined. Instead, they suggested a *neutron–balance* model in which the effective multiplication factor is defined as the ratio of the neutron production rate divided by the neutron loss rate. This contrasts with the definition of k_{eff} in the life–cycle model being the ratio of the neutron populations in successive generations. In the neutron-balance model, the neutron lifetime is defined such that the total neutron population divided by the lifetime, N/τ, is equal to the total loss rate due to absorption and leakage. When defined in this manner, the life-cycle model and the neutron-balance model yield consistent equations that describe the time rate of change of the total neutron population. Furthermore, Weinberg and Wigner showed that the neutron lifetime was merely the "*local* reciprocal lifetime,"

$$\frac{1}{\tau(\boldsymbol{r}, E)} = v\Sigma_a - v\phi^{-1} div D grad \phi \quad , \tag{7}$$

averaged over the reactor system by way of

$$\frac{1}{\tau} = \frac{\int \frac{\phi(\boldsymbol{r}, E)}{\tau(\boldsymbol{r}, E)}\, dV dE}{\int \phi(\boldsymbol{r}, E)\, dV dE} \quad . \tag{8}$$

However, without any supporting explanation, Weinberg and Wigner quickly stated that "the *effective* reciprocal lifetime to be used with the criticality factor (i.e., k_{eff}) is not the quantity defined above, but is the average of $1/\tau(\boldsymbol{r},E)$ for all neutrons weighted with their importance."

$$\frac{1}{\tau} = \frac{\int \frac{\phi^+(\boldsymbol{r},E)\phi(\boldsymbol{r},E)}{\tau(\boldsymbol{r},E)}\, dVdE}{\int \phi^+(\boldsymbol{r},E)\phi(\boldsymbol{r},E)\, dVdE} \quad . \tag{9}$$

In several places in their book, Weinberg and Wigner referred to τ as being both the neutron generation time and the neutron lifetime. They interpreted τ to be "the average time between successive generations," or "the average time between birth and death of a neutron." To complete their discussion on neutron lifetimes, they went on to derive a more general expression for a neutron lifetime based on the probability that a neutron of the i^{th} type will produce neutrons of the j^{th} type. That is,

$$\tau = \frac{\sum t_{ji} P_j^{\cdot\cdot} P_i}{\sum P_j^* P_j} \quad , \tag{10}$$

where $P \approx \nu\Sigma_f\phi$, P^* is the neutron importance, and t_{ji} is the time from birth-to-event.

In 1959, Lewins[13] published a manuscript in which he defined the neutron lifetime as "the mean time before one neutron is destroyed" and the neutron generation time as "the mean time before one neutron generates one prompt neutron or one precursor." Quantitatively, Lewins defined the neutron lifetime as

$$\tau = \frac{1}{\nu\Sigma_a + \nu DB^2} \quad , \tag{11}$$

and

$$\Lambda = \frac{1}{\nu\bar{\nu}_t\Sigma_f} \quad . \tag{12}$$

Furthermore, he recognized that in a critical reactor, the neutron production rate was equal to the neutron loss rate; hence, τ must be equal to Λ in a critical system.

In 1963, Hishihara and Ukai[14] derived an integral version of the Weinberg and Wigner[12]

expression for the effective neutron lifetime [see Eq. (10)] starting from the general neutron transport equation. They further demonstrated that their integral equation could be rewritten in a form identical to the equation previously derived by Ussachoff.[11] They, however, defined the effective neutron lifetime "as the mean interval of successive fission events in the course of the importance transport."

In 1964, Asaoka et al.[15] were developing Monte Carlo techniques to estimate the neutron lifetime and generation time using the Monte Carlo code TIMOC. In their work they defined three different neutron lifetimes: 1) the *effective* mean lifetime, 2) the *dynamical* mean lifetime, and 3) the *chronological* mean lifetime. The effective mean lifetime was defined as "the total importance divided by the rate of destruction of importance"; the dynamical mean lifetime was defined as "the total number of neutrons divided by the destruction rate"; and the chronological mean lifetime was defined as "the average time from birth to death of the neutrons." They also defined three neutron generation times analogous to the aforementioned lifetimes but based on the production of neutrons rather than on the destruction of neutrons.

In 1967, Rief and Kschwendt[16] presented two expressions used in another Monte Carlo code, MOCA, to calculate the neutron *production* lifetime and the neutron *destruction* lifetime.[c] Both of these equations were of the same general form as Eq. (10) developed by Weinberg and Wigner,[12] but with unity weighting rather than importance weighting. They found that their destruction lifetime agreed exactly with the reciprocal of the unweighted loss rate (absorption plus leakage), but that their production lifetime did not always agree with the reciprocal importance-weighted average loss rate. In 1971, Nelson[17] reviewed the work of Rief and Kschwendt and demonstrated that the production lifetime and the reciprocal importance-weighted average loss rate are similar, but that they are not precisely the same.

During the 70s, 80s and 90s, the effort to clearly define a neutron lifetime and a neutron generation time continued. In fact, one debate started in the early 80s is still drawing attention. In 1981, Marotta[18,19] proposed a new parameter, which he named *excess time*, which he claimed would be useful in "giving physical insight into the degree of utilization of neutrons toward a chain reacting process in a complicated fissionable system." The excess time was defined to be the difference between the neutron lifetime and the neutron generation time where, according to Marotta's definitions, the neutron lifetime is "the average life span of a neutron in the system until it escapes from the system or is absorbed," and the neutron generation time is "the average time between neutron generations." Marotta argued that the neutron lifetime and the neutron generation time are "numerically equal *only* for the maximum (not necessarily critical) k_{eff} state of the system." This, of course, was contrary to the popularly-held belief that the neutron lifetime and the neutron generation time are equal at delayed critical. Marotta based this conclusion on results generated by the Monte Carlo code KENO-IV which calculates both a *system* lifetime and a neutron generation time. Lewins,[20] feeling somewhat responsible for a certain amount of confusion

[c] It is presumed that the expressions presented by Rief and Kschwendt were similar, if not exact, to the expressions used by Asaoka since Rief was one of the co-authors on Asaoka paper.

concerning the neutron generation time he defined in an earlier work,[13] attempted to clarify his previous work by redefining the neutron generation time as the "mean time for one neutron to cause fission," and introducing the neutron *reproduction* lifetime as the "mean time for one neutron to be replaced by another neutron on fissioning." In his response to Marotta's claims, Lewins demonstrated that the neutron lifetime had to be equal to the neutron reproduction lifetime at delayed critical. In 1995, a third party joined into the debate. Hayashi[21] wrote a Letter to the Editor of Nuclear Science and Engineering in which he claimed that there was no difference whatsoever between the neutron lifetime, the neutron generation time, and the mean fission lifetime.

Are you confused yet? We are!

Intent

The primary purpose of this work is to develop both deterministic and Monte Carlo algorithms that clearly define a neutron lifespan, a neutron lifetime, and a neutron generation time. In addition, we will give verbal definitions that describe the physical significance of each of these quantities. When appropriate, we will link our definitions to the definitions given by previous authors. From a more global perspective, we hope that this work will eventually lead to a unified set of equations and to a more consistent terminology concerning neutron lifetimes.

II. NOMENCLATURE

In any given system, neutrons are lost from that system in one of three ways—absorption leakage, or radioactive decay. We can usually ignore radioactive decay since the half-life of a free neutron is approximately 10.4 minutes and the typical lifetime of a neutron in most systems is on the order of a few milliseconds or less. So, from a practical standpoint, there are only two ways that a neutron can be lost from a system—absorption and leakage.

It is worthwhile at this point to clearly define what we mean by a neutron *absorption* since an absorption event can be defined in several different ways. For the most part, the terminology used in the majority of the nuclear engineering textbooks is relatively consistent when defining neutron cross sections. In particular, the total macroscopic cross section, Σ_t, is defined as the sum of the macroscopic cross sections of all possible interactions. We can group these various interactions into two broad categories corresponding to a macroscopic absorption cross section, Σ_a, and a macroscopic scattering cross section, Σ_s,

$$\Sigma_t = \Sigma_a + \Sigma_s \quad , \tag{13}$$

where the scattering cross section is defined as the sum of the elastic, (n,n), and inelastic scatter-

ing, $(n, n'\gamma)$, cross sections plus any other reaction in which a single neutron is re-emitted along with any other outgoing particle produced during the scattering interaction, such as $(n, n'\alpha)$.

$$\Sigma_s = \Sigma_n + \Sigma_{n'\gamma} + \Sigma_{n'\alpha} + \ldots \quad , \tag{14}$$

The macroscopic absorption cross section includes everything else; that is, the absorption cross section is the sum of the cross sections of all non-scattering events (e.g., reactions that produce zero neutrons or more than one neutron).

$$\Sigma_a = \Sigma_\gamma + \Sigma_p + \Sigma_\alpha + \ldots + \Sigma_f + \Sigma_{2n} + \Sigma_{3n} + \ldots \quad , \tag{15}$$

where

Σ_γ is the macroscopic radiative capture cross section, (n,γ),
Σ_p is the macroscopic proton production cross section, (n,p),
Σ_α is the macroscopic alpha-particle production cross section, (n,α),
Σ_f is the macroscopic fission cross section, (n,f),
Σ_{2n} is the macroscopic $(n,2n)$ cross section,
Σ_{3n} is the macroscopic $(n,3n)$ cross section, and so forth.

For convenience, we can further separate the absorption cross section into two subcategories—parasitic[d] capture, Σ_{pc}, and neutron production reactions, Σ_π. The parasitic capture cross section is the sum of all reactions in which zero neutrons are re-emitted after the initial neutron absorption—(n,γ), (n,p), (n,α), (n,t), $(n,{}^3he)$, etc. The neutron production cross section is the sum of all reactions in which more than one neutron is re-emitted following the initial neutron absorption—(n,f), $(n,2n)$, $(n,3n)$, etc. So, we can define the absorption cross section as $\Sigma_a = \Sigma_{pc} + \Sigma_\pi$, and rewrite the total cross section as the sum of three terms.

$$\Sigma_t = \Sigma_{pc} + \Sigma_s + \Sigma_\pi \quad . \tag{16}$$

By defining the total cross section in the above manner, we now have three broad categories—reactions in which a neutron is absorbed and no neutrons are re-emitted; reactions in which a neutron is absorbed and a single neutron is re-emitted; and reactions in which a neutron is absorbed and more than one neutron is re-emitted. This categorization may seem somewhat arbi-

d. Referred to as *pure* capture by Bell.[22]

trary, but, as will be discussed in a later section, actually proves to be very useful from the standpoint of tracking the neutron population.

III. LIFESPAN, LIFETIME, AND GENERATION TIME

In this section, we define, by way of example, three different types of time constants associated with the life of a neutron in a given system—a neutron lifespan, a removal lifetime, and a neutron generation time.

Consider a multiplying system with an effective multiplication factor of $k_{eff} = 0.9$ that is in source equilibrium with an external/intrinsic neutron source. Neutrons will disappear from this system in one of two ways—absorption or leakage. As noted in the previous section, when a neutron is absorbed, it can be absorbed in either a neutron production reaction or a parasitic capture reaction. It can be readily shown that in any multiplying system the fraction of neutrons absorbed in a fission reaction is equal to $k_{eff}/\bar{\nu}_t$ where $\bar{\nu}_t$ is the average number of neutrons (prompt plus delayed) released per fission. Therefore, if we assume that $\bar{\nu}_t = 2.5$ for this hypothetical system, then 36% of the neutrons will be absorbed in a fission reaction. The remaining 64% will be lost by leakage or parasitic capture—of which we arbitrarily assume, for purposes of this example, that 20% are absorbed in a parasitic capture, and the remaining 44% leak from the system. We also assume that the average time from birth–to–fission of those neutrons that are destined to be absorbed in a fission reaction is 100 μs, the average time from birth–to–absorption of those neutrons that are destined to be absorbed in a parasitic capture reaction is 90 μs, and the average time from birth–to–leakage of those neutrons destined to leak from the system is 50 μs. Consequently, a neutron in this hypothetical system will live, on an average, for

$$\tau = 0.36 \times 100 + 0.20 \times 90 + 0.44 \times 50 = 76\ \mu s \quad .$$

Let's also assume that the intrinsic source strength is such that the equilibrium neutron population in this system is 1.0; that is to say, if a snap shot were taken at any point in time, the total number of neutrons that could be found in this system would be, on an average, 1.0. To maintain this equilibrium neutron population, one neutron must be injected into the system via the fixed source and the fission source once every 76 μs since one neutron is lost from the system once every 76 μs. This equilibrium neutron population and average removal lifetime corresponds to an average loss rate of 13,158 n/s—of which 4,737 n/s (i.e., 36%) are being absorbed in fission reactions, 2,632 n/s (i.e., 20%) are being parasitically captured, and 5,789 n/s (i.e., 44%) are leaking.

Because the system is in source equilibrium at a k_{eff}=0.9, neutrons must be appearing in the system at a rate of 13,158 n/s —of which $Sk_{eff}/(1 - k_{eff})$ neutrons per second are being pro-

duced by fission reactions. Hence, 11,842 n/s are being produced by fissions while the remaining 1,316 n/s are being injected by the source. This, of course, is consistent with the neutron loss rate in which 4,737 neutrons are absorbed in a fission reaction, each yielding 2.5 neutrons/second.

Using this example we can readily identify several neutron time constants of primary interest in reactor physics. In the context of neutron-balance theory, a *neutron lifespan* will be used to denote the mean time from birth-to-event, whereas, a *neutron lifetime* is defined as the average time from event-to-event. First we note that the mean time between the loss of a neutron from the system (event-to-event) is 76 μs. We define this quantity as the mean *removal* lifetime. It is simply the average of the individual *lifespans* corresponding to fission, parasitic absorption, and leakage, weighted by their respective probability of occurrence. The lifespans, in turn, are defined as the average time from birth to a specific event. In our example, the average lifespan for fission is 100 μs; the average lifespan for parasitic absorption is 90 μs; and the average lifespan for leakage is 50 μs.

Note that these lifespans, however, do not represent the mean time between events of a particular type; the mean time between fission events corresponds to 211.1 μs (i.e., 1/4,737), the mean time between parasitic absorptions corresponds to 380 μs (i.e., 1/2,632), and the mean time between leakage events corresponds to 173 μs (i.e.,1/5,789). To our knowledge, this concept of a neutron lifespan and its clear distinction from a neutron lifetime has not previously appeared in the literature. This distinction, however, should be carefully noted, particularly when interpreting lifetime calculations from Monte Carlo codes. MCNP-4A,[23] for example, calculates an unweighted removal lifetime, and three different neutron lifespans—escape, fission, and capture.[e] The KENO[24] code, on the other hand, calculates a removal lifetime (which is referred to as the system lifetime) and another quantity that closely resembles an unweighted fission lifetime.

The neutron lifespan and neutron removal lifetime, as defined above, characterize the loss rate from the system. We can also define another lifetime that is associated with the production of neutrons. We refer to this lifetime as the neutron generation time and define it as the mean time between the appearance of fission neutrons. Hence, in our example, the neutron generation time corresponds to 84.5 μs (i.e., 1/11,842). It is very important to recognize that the mean time between the appearance of a fission neutron is a factor of $\overline{\nu}$ smaller than the mean time between fission events, 211.1 μs. By defining the neutron generation time in this manner, we can then state that the effective multiplication factor is the ratio of the neutron lifetime to the neutron generation time. Had we included the contribution from the fixed source in our definition of the neutron generation time, this statement would not be true; in equilibrium, the mean time between the appearance of either a fission or source neutron would be identically equal to the mean time between the loss of neutrons from the system (i.e., 76 μs), so the ratio would always be 1.0. In the absence of a source, if neutrons are being lost from a system at a rate that is faster than the rate at which they are being produced, the system must be subcritical; and if neutrons are being produced at a rate

[e.] The capture life span in MCNP actually corresponds to the absorption life span as defined in this manuscript while the fission life span is merely a subset of that quantity.

that is faster than they are being lost, then the system must be supercritical; when the two rates are equal, the system must be exactly critical.

In summary, we can state that a neutron life span is the average time from birth–to–event, whereas, a neutron lifetime is the mean time from event–to–event in which the event is either an absorption or a leakage. A neutron generation time, on the other hand, is the mean time between the appearance of the neutrons produced by fission, (*n,2n*), ...etc. reactions. As we have demonstrated, albeit somewhat informally, the neutron lifetime is a function of the individual neutron life spans. The neutron generation time, however, is strictly a function of the fission, (*n,2n*), ...etc. reaction rates and, as such, is not related to the neutron lifetime per se. However, the neutron lifetime and the neutron generation time can be related to each other by the effective multiplication factor.

IV. TIME-DEPENDENT TRANSPORT EQUATION

Consider a hypothetical system that is infinite in extent and contains a hypothetical material that only scatters neutrons (i.e., $\Sigma_t = \Sigma_s$). If a million neutrons are injected into this system at t=0, and we counted neutrons at some later time, we would expect to find all one million neutrons to be still in the system. The scattering reactions that would be constantly occurring in the system may cause drastic changes in the angular, energy, and spatial distribution of those neutrons, but these scatterings reactions would not cause a *net* change in the neutron population.

If this same hypothetical material, however, had a small (*n,2n*) cross section associated with it (i.e., $\Sigma_t = \Sigma_s + \Sigma_{2n}$), we would expect to find more than one million neutrons in the system at a later time; every time a neutron is *absorbed* in a (*n,2n*) reaction, two neutrons would be re-emitted—thus, there would be a net gain of one neutron per (*n,2n*) reaction. Similarly, if the material were capable of fissioning, then there would be, on an average, $\bar{\nu}$ neutrons released per neutron absorbed in a (*n,f*) reaction, which would result in a net gain of $\bar{\nu} - 1$ neutrons in the neutron population.

In contrast, if this hypothetical scattering material were incapable of multiplying, but exhibited some parasitic capture reactions (i.e., $\Sigma_t = \Sigma_s + \Sigma_{pc}$), then each time a neutron was absorbed in a parasitic capture, there would be a net decrease of one neutron in the neutron population. Consequently, the neutron population would decrease with time and would eventually decay to zero.

In general, we can describe the time-dependent behavior of the total neutron population for any of these situations using the time-dependent transport equation, where we include the additional loss rate associated with neutrons that can potentially leak from a finite system. If we assume that all neutrons in the system have equal importance (which, as will be discussed in a later section, is not true in a multiplying system) and that the cross sections are time independent, the energy–, spatial–, and time–dependent angular flux can be written as,[22]

$$\frac{1}{\mathrm{v}} \cdot \frac{\partial \Psi}{\partial t} + \Omega \cdot \nabla \Psi + \Sigma_t \Psi = \int \Sigma'_s \Psi' d\Omega' dE' + \int f_\pi \Sigma'_\pi \Psi' d\Omega' dE' + Q \quad , \tag{17}$$

where

$$\begin{aligned}
\Psi &= \Psi(\boldsymbol{r}, \Omega, E, t) \quad , \\
\Sigma_t &= \Sigma_t(\boldsymbol{r}, \Omega, E) \quad , \\
\Sigma'_s &= \Sigma'_s(\boldsymbol{r}; \Omega', E' \rightarrow \Omega, E) \quad , \\
f_\pi &= f_\pi(\boldsymbol{r}; \Omega', E' \rightarrow \Omega, E) \quad , \\
\Sigma'_\pi &= \Sigma'_\pi(\boldsymbol{r}; \Omega', E' \rightarrow \Omega, E) \quad , \text{ and} \\
Q &= Q(\boldsymbol{r}, \Omega, E, t) \quad ,
\end{aligned}$$

in which the meaning of the macroscopic cross sections have been previously described, v is the neutron velocity, and f_π is the probability that if a neutron of direction Ω' and energy E' has a collision of production type π, there will emerge from the collision a neutron in the direction interval $d\Omega$ about Ω with energy in dE about E. For fission events, f_π is normalized to $\bar{\nu}_t$. That is,

$$\int f_f(\boldsymbol{r}; \Omega', E' \rightarrow \Omega, E) d\Omega dE = \bar{\nu}_t \quad , \tag{18}$$

and for $(n,2n)$ production reactions,

$$\int f_{2n}(\boldsymbol{r}; \Omega', E' \rightarrow \Omega, E) d\Omega dE = 2 \quad , \tag{19}$$

and so forth for $(n,3n)$, …, etc. reactions. (To simplify our nomenclature somewhat, we have used a single integral symbol to represent multiple integrals over the differential variables listed in the integrand. This simplification in nomenclature shall be used throughout the remainder of this manuscript.)

Equation (17) is merely a statement of neutron conservation as applied to an infinitesimal element of volume, direction, and energy. If it is integrated over all directions, it becomes a statement of neutron conservation for a small element of volume and energy written in terms of the total neutron flux, ϕ, and the neutron current, $\boldsymbol{J}$.

$$\frac{1}{\mathrm{v}} \cdot \frac{\partial \phi}{\partial t} + \nabla \bullet \boldsymbol{J} + \Sigma_t \phi = \int \Sigma'_s \phi' dE' + \int \chi_f \bar{\nu}_t \Sigma'_f \phi' dE' + \int \chi_{2n} 2 \Sigma'_{2n} \phi' dE' + \ldots + Q \quad , \tag{20}$$

where the total flux and the neutron current are defined as

$$\phi = \phi(\boldsymbol{r}, E, t) = \int \Psi d\Omega \quad ,$$

$$\boldsymbol{J} = \boldsymbol{J}(\boldsymbol{r}, E, t) = \int \Omega \Psi d\Omega \quad ,$$

and

$$\begin{aligned}
&\Sigma_t = \Sigma_t(\boldsymbol{r}, E) \quad , \\
&\Sigma'_s = \Sigma'_s(\boldsymbol{r}; E' \rightarrow E) \quad , \\
&\Sigma'_f = \Sigma'_f(\boldsymbol{r}; E' \rightarrow E) \quad , \\
&\chi_f = \chi_f(E) \quad , \\
&\chi_{2n} = \chi_{2n}(E) \quad , \text{and} \\
&Q = Q(\boldsymbol{r}, E, t) \quad .
\end{aligned}$$

Integration of Eq. (20) over a finite region of volume and energy now yields a conservation equation for the entire neutron population in the region.

$$\int \frac{1}{v} \cdot \frac{\partial \phi}{\partial t} dV dE + \int \nabla \bullet \boldsymbol{J} \; dV dE + \int \Sigma_t \phi \; dV dE = \int \Sigma'_s \phi' dE' \; dV dE +$$

$$\int \chi_f \bar{\nu}_t \Sigma'_f \phi' dE' dV dE + \int \chi_{2n} 2\Sigma'_{2n} \phi' dE' dV dE + \ldots + \int Q \; dV dE \quad . \tag{21}$$

From the definition of the total cross section [see Eq. (13)], we note

$$\int \Sigma_t \phi \; dV dE - \int \Sigma'_s \phi' dE' \; dV dE = \int \Sigma_a \phi \; dV dE \quad , \tag{22}$$

where we again stress that the absorption cross section, Σ_a , includes all reactions that are non-scattering type. Hence, we can rewrite Eq. (21) as

$$\frac{dN}{dt} = P - L + S \quad , \tag{23}$$

where N is the total, *unweighted* neutron population,

$$N = \int \frac{\phi}{v} \cdot dVdE \quad , \tag{24}$$

P represents the unweighted neutron production rate,

$$P = \int \chi_f \bar{\nu}_t \Sigma'_f \phi' dE' dVdE + \int \chi_{2n} 2\Sigma'_{2n} \phi' dE' dVdE + \ldots \quad , \tag{25}$$

L represents the unweighted neutron lost rate due to leakage and absorption,

$$L = \int \nabla \bullet \boldsymbol{J} \; dVdE + \int \Sigma_a \phi \; dVdE \quad , \tag{26}$$

and S is the unweighted fixed source rate.

If the neutron production rate is greater than the neutron loss rate, the total neutron population will increase with time and, once the shape factor has reached its asymptotic distribution, will increase at an exponential rate.

$$N = N_o e^{\alpha t} \quad . \tag{27}$$

If we neglect the source term and we insert Eq. (27) into Eq. (23), we obtain the following time-dependent equation,

$$\alpha N = P - L \quad . \tag{28}$$

As a purely arbitrary choice, we divide both sides of Eq. (28) through by the loss rate. This yields an equation of the form

$$\alpha \tau_s = \kappa - 1 \quad , \tag{29}$$

where we define an instantaneous multiplication factor, κ, as

$$\kappa = \frac{\int \chi_f \bar{\nu}_t \Sigma'_f \phi' dE' dV dE + \int \chi_{2n} 2\Sigma'_{2n} \phi' dE' dV dE + \ldots}{\int \nabla \bullet \boldsymbol{J}\, dV dE + \int \Sigma_a \phi\, dV dE} \quad , \tag{30}$$

and an unweighted system lifetime, τ_s, as

$$\tau_s = \frac{N}{\int \nabla \bullet \boldsymbol{J}\, dV dE + \int \Sigma_a \phi\, dV dE} \quad . \tag{31}$$

The instantaneous multiplication factor, κ, is the total neutron production rate (from all reactions that produce more than *one* neutron) per neutron lost from the system by leakage and absorption. This particular multiplication factor is not the traditional *k*–eigenvalue used in reactor physics;[22] *however, it is very similar.* In the traditional *k*-eigenvalue formulation, the neutron production terms associated with (*n,2n*), (*n,3n*), …, etc. are usually accounted for adding $2\Sigma_{2n}$, $3\Sigma_{3n}$, etc. to the elastic and inelastic scattering cross sections, Σ_s, so that the *k*-eigenvalue solution is based solely on the production of fission neutrons. The *k*–eigenvalue, thus, satisfies the pseudo-stationary transport equation written as

$$\int \nabla \bullet \boldsymbol{J}\, dV dE + \int (\Sigma_a - 2\Sigma_{2n} - \ldots)\phi\, dV dE = \frac{1}{k}[\int \chi_f \bar{\nu}_t \Sigma'_f \phi' dE' dV dE] \quad . \tag{32}$$

When written in the above form, we note that the traditional *k*–eigenvalue corresponds to the number of fission neutrons produced per neutron lost by leakage and absorption *minus those that are absorbed in (n,2n), (n,3n), ..., etc. multiplied by their respective multiplicity.* Fortunately, in most thermal reactors, the threshold energy required to produce (*n,2n*), (*n,3n*), …, etc. reactions is well above the energy of most fission neutrons. Consequently, the absorption cross sections for those particular reactions are negligible relative to the total absorption cross section; hence, κ and *k* are nearly identical providing the *k*-eigenfunctions and the α-eigenfunctions are similar—which may not always be the case. As shown by several authors, when the system is not in the vicinity of delayed critical, the spectrum of the eigenfunctions obtained from an α-eigenvalue solution can differ significantly from the spectrum obtained from a *k*-eigenvalue solution. Unlike thermal systems, the (*n,2n*), (*n,3n*), …, etc. reactions in fast systems can make a discernible contribution. Using the Godiva system as a test case, Robert Little[31] found that k_{eff} could be altered between 0.0024 and 0.0049 Δk depending on the exact method used to fold Σ_{2n} and Σ_{3n} into the cross section set.

(As an aside, if a new κ-eigenvalue problem is formulated in accordance with Eq. (30),

the effective delayed-neutron fraction that appears in the point-kinetic model must also be modified in order to maintain consistency. The value of β_i for each delayed neutron group must be lowered somewhat to account for the fact that not all neutrons produced in the system leave behind a trail of fission fragments that can decay by delayed neutron. This can be easily accomplished by defining a reduced delayed neutron fraction as $\beta'_i = \beta_i F$ where F is given by

$$F = \frac{\int \chi_f \bar{\nu}_t \Sigma'_f \phi' dE' dV dE}{\int \chi_f \bar{\nu}_t \Sigma'_f \phi' dE' dV dE + \int \chi_{2n} 2\Sigma'_{2n} \phi' dE' dV dE + \ldots} \quad .)$$

Before proceeding, we would like to emphasis that the decision to subtract the integral of the scattering reaction rate from the integral of the total reaction rate to obtain an integral absorption rate was purely arbitrary. Furthermore, the decision to divide through by the sum of the leakage rate and the absorption rate was also arbitrary. Had we chosen to retain the integral of the scattering reaction rate on the right-hand side of Eq. (21) and divide through by the sum of the leakage rate and the total reaction rate, we would obtain

$$\alpha \tau_{coll} = \gamma - 1 \quad , \tag{33}$$

where

$$\gamma = \frac{\int \Sigma'_s \phi' dE' \, dV dE + \int \chi_f \bar{\nu}_t \Sigma'_f \phi' dE' dV dE + \int \chi_{2n} 2\Sigma'_{2n} \phi' dE' dV dE + \ldots}{\int \nabla \bullet \boldsymbol{J} \, dV dE + \int \Sigma_t \phi \, dV dE} \quad , \tag{34}$$

and

$$\tau_{coll} = \frac{N}{\int \nabla \bullet \boldsymbol{J} \, dV dE + \int \Sigma_t \phi \, dV dE} \quad . \tag{35}$$

Equation (34) is, by definition, the instantaneous value of the γ-eigenvalue defined by Ronen,[25] which is referred to as the effective *collision* multiplication factor.[30] From a physical standpoint, γ represents the total neutron production rate (including the neutrons re-emitted following a scattering interaction) divided by the total interaction rate of neutrons within the system (including leakage). As with the *k*-eigenvalue, the γ-eigenvalue is just another measure of the state of a multiplying system; when $\gamma<1$, the system is subcritical; when $\gamma=1$, the system is critical; and, when $\gamma>1$, the system is supercritical. However, unlike the *k*-eigenvalue, γ is not zero when the fission source is zero. Even though the numerical value of γ-eigenvalue differs from the *k*-eigenvalue at a particular system configuration, it is just as meaningful and just as useful in characteriz-

ing the state of a multiplying system.

The collision lifetime, τ_{coll}, defined by Eq. (35) represents the mean time per unit neutron population between interactions of all types (leakage plus collision), whereas, the system lifetime defined by Eq. (31) represents the mean time per unit neutron population between neutron losses from the system due to leakage and absorption (i.e., non-scattering events). Obviously, the collision lifetime can be significantly smaller than the system lifetime since numerous scattering events can occur prior to a neutron being removed from the system due to an absorption or leakage. Since the α in Eqs. (28) and (33) must be the same, it follows that

$$\frac{\tau_{coll}}{\tau_s} = \frac{\gamma - 1}{\kappa - 1} \approx \frac{\gamma - 1}{k - 1} \quad . \tag{36}$$

In an analogous fashion, we could just as easily (and just as arbitrarily) move the integral of the total reaction rate to the right-hand side of Eq. (21) and divide through by the integral of the leakage rate. This would lead to

$$\alpha\tau_l = \delta - 1 \quad , \tag{37}$$

where

$$\delta = \frac{\int \Sigma'_s \phi' dE' \, dV dE + \int \chi_f \bar{\nu}_t \Sigma'_f \phi' dE' dV dE + \ldots - \int \Sigma_t \phi \, dV dE}{\int \nabla \bullet \boldsymbol{J} \, dV dE} \quad , \tag{38}$$

and

$$\tau_l = \frac{N}{\int \nabla \bullet \boldsymbol{J} \, dV dE} \quad . \tag{39}$$

Equation (38) is, by definition, the instantaneous value of the δ-eigenvalue,[25-29] which is referred to as the effective *density* factor.[27,30] As with the *k*-eigenvalue and the γ-eigenvalue, the δ-eigenvalue is just another measure of the state of a multiplying system; when $\delta<1$, the system is subcritical; when $\delta=1$, the system is critical; and, when $\delta>1$, the system is supercritical. Similar to the γ-eigenvalue, δ is not zero when the fission rate is zero. And finally, the leakage lifetime, τ_l, defined by Eq. (39) represents the mean time per unit neutron population between leakage events.

Because the vast majority of the nuclear industry uses the *k*-eigenvalue as their measure of the degree of departure from a critical condition, we will confine the remainder of this manuscript to defining various other reaction-rate lifetimes in terms of the system lifetime.

V. GENERAL DEFINITION OF A NEUTRON LIFETIME

From Eq. (31), we note that the unweighted system lifetime is the unweighted neutron population divided by the rate at which neutrons are being lost from the system via leakage and absorption. That is,

$$\frac{N}{\tau_s} = \int \nabla \bullet \boldsymbol{J} \; dVdE + \int \Sigma_a \phi \; dVdE \quad . \tag{40}$$

In general, we can define any number of unweighted neutron lifetimes corresponding to any arbitrary type of reaction as

$$\frac{N_j}{\tau_i} \equiv I_i \quad , \tag{41}$$

where N_j is the total neutron population in volume V_j, and I_i is an interaction rate or the sum of several interaction rates representing a particular process. For example, we can rewrite Eq. (40) in terms of a leakage lifetime, τ_l, and an absorption lifetime, τ_a, as

$$\frac{N}{\tau_s} = \frac{N}{\tau_l} + \frac{N}{\tau_a} \quad , \tag{42}$$

where

$$\frac{N}{\tau_l} = \int \nabla \bullet \boldsymbol{J} \; dVdE \quad , \tag{43}$$

and

$$\frac{N}{\tau_a} = \int \Sigma_a \phi \; dVdE \quad . \tag{44}$$

From Eq. (42), we note that the reciprocal of the system lifetime is the sum of the reciprocals of the leakage lifetime and the absorption lifetime (i.e., the sum of the harmonics).

$$\frac{1}{\tau_s} = \frac{1}{\tau_l} + \frac{1}{\tau_a} \quad . \tag{45}$$

The leakage lifetime can be related more directly to the system lifetime by noting that the ratio of the leakage rate to the total loss rate is, by definition, the probability of a leakage event,

p_l .

$$\frac{\tau_s}{\tau_l} = \frac{\int \nabla \bullet \boldsymbol{J}\ dVdE}{\int \nabla \bullet \boldsymbol{J}\ dVdE + \int \Sigma_a \phi\ dVdE} = p_l \quad . \tag{46}$$

Hence,

$$\tau_l = \frac{\tau_s}{p_l} \quad . \tag{47}$$

In a similar fashion, the absorption lifetime can be related to the total system lifetime by taking the ratio of the absorption rate to the total loss rate. This leads to,

$$\frac{\tau_s}{\tau_a} = \frac{\int \Sigma_a \phi\ dVdE}{\int \nabla \bullet \boldsymbol{J}\ dVdE + \int \Sigma_a \phi\ dVdE} = p_a \quad , \tag{48}$$

where p_a is the probability of a neutron absorption. Upon rearrangement, we obtain

$$\tau_a = \frac{\tau_s}{p_a} \quad . \tag{49}$$

When Eqs. (47) and (49) are inserted back into Eq. (45), we obtain an equality since the sum of the probabilities of leakage and absorption must, by definition, add to 1.0 [see Eqs. (46) and (48)].

We can also express the leakage lifetime and the absorption lifetime in terms of other constituent lifetimes. For example, the absorption rate is comprised of numerous types of parasitic captures and several types of neutron production reactions [see Eq. (15)]. Consequently, we can write the sum of the various absorption reaction rates as

$$\frac{N}{\tau_a} = \frac{N}{\tau_f} + \frac{N}{\tau_{2n}} + \frac{N}{\tau_{3n}} + \ldots + \frac{N}{\tau_\gamma} + \frac{N}{\tau_p} + \frac{N}{\tau_\alpha} + \ldots \tag{50}$$

in which,

$$\frac{N}{\tau_f} = \int \Sigma_f \phi\ dVdE \quad , \tag{51}$$

and so forth. Each of these constituent lifetimes is related to the system lifetime as

$$\tau_f = \frac{\tau_s}{p_f} \quad , \tag{52}$$

where

$$p_f = \frac{\int \Sigma_f \phi \; dVdE}{\int \nabla \bullet \boldsymbol{J} \; dVdE + \int \Sigma_a \phi \; dVdE} \quad , \tag{53}$$

and so forth.

As before, we can state that the reciprocal of the absorption lifetime is the sum of the harmonics of the various constituent lifetimes associated with absorption reactions.

$$\frac{1}{\tau_a} = \frac{1}{\tau_f} + \frac{1}{\tau_{2n}} + \frac{1}{\tau_{3n}} + \ldots + \frac{1}{\tau_\gamma} + \frac{1}{\tau_p} + \frac{1}{\tau_\alpha} + \ldots = \frac{p_a}{\tau_s} \quad , \tag{54}$$

where

$$p_a = p_f + p_{2n} + p_{3n} + \ldots + p_\gamma + p_p + p_\alpha + \ldots \quad . \tag{55}$$

Up to this point, we have dealt primarily with lifetimes that are associated with loss mechanisms—leakage and absorption. However, we can also define similar lifetimes associated with the production of neutrons. We begin by defining the neutron production lifetime, τ_π, as

$$\frac{N}{\tau_\pi} = \int \chi_f \bar{\nu}_t \Sigma'_f \phi' dE' dVdE + \int \chi_{2n} 2\Sigma'_{2n} \phi' dE' dVdE + \ldots \quad . \tag{56}$$

The neutron production lifetime, τ_π, is an unweighted version of the neutron generation time found in the one-region, point-kinetic model. It can be related to the unweighted system lifetime by dividing Eq. (56) by Eq. (40) and recalling the definition of the κ-eigenvalue [see Eq. (30)]. Hence,

$$\tau_\pi = \frac{\tau_s}{\kappa} \approx \frac{\tau_s}{k} \quad . \tag{57}$$

(The neutron generation time will be discussed more in a later section). We can also express the neutron production lifetime in terms of the previously defined fission lifetime, τ_f, and the (*n,2n*) lifetime, etc.

$$\frac{1}{\tau_\pi} = \frac{\bar{\nu}_t}{\tau_f} + \frac{2}{\tau_{2n}} + \ldots \quad . \tag{58}$$

VI. ELASTIC AND INELASTIC SCATTERING LIFETIMES

So far, we have concentrated mainly on lifetimes associated with reactions that remove and/or produce neutrons in multiplying and nonmultiplying systems. We have said very little about the elastic and inelastic scattering lifetimes, which can be of great interest when analyzing the multiple die-away modes that are frequently observed in nuclear well-logging measurements. For example, in the standard pulsed-neutron, well-logging technique, a short burst of 14 Mev neutrons is injected into a rock formation. The subsequent die-away of these neutrons is observed indirectly by measuring photons produced by a wide variety of neutron-induced reactions that occur in the formation. During the initial part of the die-away, the photon flux is dominated by those photons produced by inelastic scatterings in the iron casing that typically lines the bore hole. These photons die-away at a relatively fast rate and eventually disappear as the neutrons moderate and drop below the threshold energy required for inelastic scattering to occur. At late times, the photon flux is predominantly produced by photons created during the radiative capture of those neutrons that survived long enough to become thermalized. We can define a time constant associated with elastic and inelastic scattering in the same fashion as we did when we defined a system lifetime. That is,

$$\frac{N}{\tau_n} = \int \Sigma'_n \phi' dE' \, dV dE \quad , \tag{59}$$

and

$$\frac{N}{\tau_{n'}} = \int \Sigma'_{n'} \phi' dE' \, dV dE \quad . \tag{60}$$

The elastic and inelastic scattering lifetimes are also of some interest in reactor physics.

The ratio of the total scattering rate to the loss rate is a direct measure of the average number of scatterings that occur prior to a neutron being lost from the system.

$$\frac{\tau_{scat}}{\tau_s} = \frac{\int \Sigma'_n \phi' dE' \; dVdE + \int \Sigma'_{n'} \phi' dE' \; dVdE}{\int \nabla \bullet \boldsymbol{J} \; dVdE + \int \Sigma_a \phi \; dVdE} = \overline{C} \quad . \tag{61}$$

This quantity can be used as a qualitative measure of the moderating power of various core materials.

VII. ALTERNATIVE FORMS OF THE UNWEIGHTED SYSTEM LIFETIME

In the previous sections, we presented a methodology for formulating neutron lifetimes based on various reaction rates and, in all cases, we related those lifetimes to the system lifetime. The system lifetime was chosen as our reference lifetime simply because it is the lifetime that is associated with the instantaneous multiplication factor, κ, which is nearly identical to k in most systems. We could have just as easily related all of the various lifetimes to the collision lifetime [see Eq. (35)], which, as previously mentioned, is the lifetime associated with the γ-eigenvalue.

Furthermore, we wrote all of the lifetime equations in terms of the total, unweighted neutron population, N. However, when evaluating τ_s, it is much more convenient to express the total neutron population N in terms of the neutron flux distribution in the system. This is accomplished by noting that the spatial– and energy–dependent neutron density, $n(\mathbf{r},E,t)$, is related to the total neutron flux by $\phi(\mathbf{r}, E, t) = \mathrm{v} n(\mathbf{r}, E, t)$. Hence, the total neutron population in volume V can be obtained by integrating the neutron density over space and energy.

$$N = \int \frac{\phi}{\mathrm{v}} \cdot dVdE \quad . \tag{62}$$

Therefore, Eq. (31) can now be written as

$$\tau_s = \frac{\int \frac{\phi}{\mathrm{v}} \; dVdE}{\int \nabla \bullet \boldsymbol{J} \; dVdE + \int \Sigma_a \phi \; dVdE} \quad . \tag{63}$$

When written in this form, τ_s can be readily evaluated using the flux solution generated by a deterministic or Monte Carlo code.

The system lifetime defined in Eq. (63) is expressed in terms of the total loss rate due to leakage and absorption. It can also be written in terms of the total neutron production rate and the effective multiplication factor, κ. Recall from Section IV, we arbitrarily chose to divide Eq. (28) through by the loss rate, *L*. This produced an equation of the form

$$\alpha\frac{N}{L} = \frac{P}{L} - 1 \quad , \tag{64}$$

from which we defined *N*/*L* as τ_s and *P*/*L* as κ [see Eq. (29)]. However, we could just as easily have chosen to divide Eq. (28) through by the neutron production rate, *P*. This would lead to

$$\alpha\frac{N}{P} = 1 - \frac{L}{P} \quad , \tag{65}$$

which is equivalent to

$$\alpha\left(\frac{N}{P}\right)\kappa = \kappa - 1 \quad . \tag{66}$$

By comparison with Eq. (29), we note that the system lifetime, τ_s, can also be expressed as

$$\tau_s = \frac{\kappa \int \frac{\phi}{v}\, dVdE}{\int \chi_f \bar{\nu}_t \Sigma'_f \phi' dE' dVdE + \int \chi_{2n} 2\Sigma'_{2n} \phi' dE' dVdE + \ldots} \quad . \tag{67}$$

Note that when the production rates associated with (*n,2n*), (*n,3n*), etc. are small, κ becomes almost identical to *k* and Eq. (67) reduces to

$$\tau_s = \frac{k_{eff} \int \frac{\phi}{v}\, dVdE}{\int \chi_f \bar{\nu}_t \Sigma'_f \phi' dE' dVdE} \quad . \tag{68}$$

Furthermore, under steady-state conditions such as subcritical source equilibrium, we can

also write the system lifetime as a function of the neutron production rate since the total loss rate must be equal to the total production rate. Hence,

$$\tau_s = \frac{\int \frac{\phi}{v} \, dVdE}{\int \bar{\nu}_t \Sigma_f \phi \, dVdE + \int 2\Sigma_{2n}\phi \, dVdE + ... + \int \nabla \bullet \boldsymbol{J}_{in} \, dVdE + \int s \, dVdE} \quad . \tag{69}$$

where s is an intrinsic source and $\boldsymbol{J}_{in}$ is an influx of neutrons from an external source.

Using Eqs. (63) and (68), we can now derive a couple well–known expressions for the neutron lifetime in multiplying and nonmultiplying mediums. If we assume one energy group of neutrons and an infinite medium (i.e., no leakage), then Eq. (63) reduces to

$$\tau_s = \frac{1}{\bar{v}\bar{\Sigma}_a} \quad , \tag{70}$$

where

$$\bar{\Sigma}_a = \frac{\int \Sigma_a \phi \, dVdE}{\int \phi \, dVdE} \quad , \tag{71}$$

and

$$\bar{v} = \frac{\int \phi \, dVdE}{\int \frac{\phi}{v} \, dVdE} \quad . \tag{72}$$

Making these same assumptions in Eq. (68) leads to

$$\tau_s = \frac{k_{eff}}{\bar{v}(\bar{\nu}_t \bar{\Sigma}_f)} \quad , \tag{73}$$

where we have neglected all nonfission neutron production terms in the denominator.

VIII. ADJOINT-WEIGHTED SYSTEM LIFETIMES

In multiplying systems, the importance of an individual neutron is a function of its position, energy, and the direction it is traveling. This stems from the fact that a neutron born in the center of a multiplying system is less likely to leak from the system than a neutron born at the edge of the system and, thus, has a greater chance of being absorbed in a fission reaction that can further propagate a fission chain. When coupled with the fact that a neutron born in the center of the system lives longer, on an average, than a neutron born at the outer edge, the lifetime that best characterizes the dynamic behavior of a multiplying system must take into account the importance of each neutron in the system. The adjoint flux is a measure of this importance.

To derive an expression for the adjoint-weighted system lifetime, we first define the angular adjoint flux as the function that satisfies the following equation.

$$-\Omega \cdot \nabla \Phi^+(\boldsymbol{r}, \Omega, E) + \Sigma_t \Phi^+(\boldsymbol{r}, \Omega, E) =$$

$$\int \Sigma'_s \Phi^+(\boldsymbol{r}, \Omega', E') d\Omega' dE' + \int f_\pi \Sigma'_\pi \Phi^+(\boldsymbol{r}, \Omega', E') d\Omega' dE' \quad , \tag{74}$$

where

$$\Sigma_t = \Sigma_t(\boldsymbol{r}, \Omega, E) \quad ,$$
$$\Sigma'_s = \Sigma'_s(\boldsymbol{r};\Omega, E \rightarrow \Omega', E') \quad ,$$
$$f_\pi = f_\pi(\boldsymbol{r};\Omega, E \rightarrow \Omega', E') \quad , \text{and}$$
$$\Sigma'_\pi = \Sigma'_\pi(\boldsymbol{r};\Omega, E \rightarrow \Omega', E') \quad .$$

If we multiply Eq. (20) by Φ^+ and we integrate over angle, energy, and space, we obtain an equation of the form

$$\frac{dN^+}{dt} = P^+ - L^+ + S^+ \quad , \tag{75}$$

where N^+ is the total, adjoint-weighted neutron population, defined as

$$N^+ = \int \frac{\Phi^+ \Psi}{v} \, d\Omega dV dE \quad , \tag{76}$$

P^+ represents the adjoint-weighted neutron production rate, defined as

$$P^+ = \int \Phi^+ \chi_f \bar{\nu}_t \Sigma'_f \Psi' \, d\Omega dE' dV dE + \int \Phi^+ \chi_{2n} 2\Sigma'_{2n} \Psi' \, d\Omega dE' dV dE + \ldots \tag{77}$$

L^+ represents the adjoint-weighted lost rate due to leakage and absorption, defined as

$$L^+ = \int \Phi^+ \nabla \bullet \boldsymbol{J} \, d\Omega dV dE + \int \Phi^+ \Sigma_a \Psi \, d\Omega dV dE \quad , \tag{78}$$

and S^+ is the adjoint-weighted source rate.

As before, if P^+ is not equal to L^+ and the shape factor has reached its asymptotic distribution, the total, adjoint-weighted neutron population will increase (or decrease) at an exponential rate.

$$N^+ = N_o^+ e^{\alpha^+ t} \quad . \tag{79}$$

If we neglect the source term and we insert Eq. (79) into Eq. (75), we obtain an equation of the form,

$$\alpha^+ N^+ = P^+ - L^+ \quad . \tag{80}$$

Again, by arbitrary choice, we divide both sides of Eq. (80) through by the loss rate. This yields,

$$\alpha^+ \tau_s^+ = \kappa^+ - 1 \quad , \tag{81}$$

where κ^+ is given by

$$\kappa^+ = \frac{\int \Phi^+ \chi_f \bar{\nu}_t \Sigma'_f \Psi' \, d\Omega dE' dV dE + \int \Phi^+ \chi_{2n} 2\Sigma'_{2n} \Psi' \, d\Omega dE' dV dE + \ldots}{\int \Phi^+ \nabla \bullet \boldsymbol{J} \, d\Omega dV dE + \int \Phi^+ \Sigma_a \Psi \, d\Omega dV dE} \quad , \tag{82}$$

and the adjoint-weighted system lifetime is defined as

$$\tau_s^+ = \frac{\int \frac{\Phi^+\Psi}{v}\, d\Omega dV dE}{\int \Phi^+ \nabla \bullet \boldsymbol{J}\, d\Omega dV dE + \int \Phi^+ \Sigma_a \Psi\, d\Omega dV dE} \quad , \tag{83}$$

Alternatively, we can just as easily divide Eq. (80) through by the adjoint-weighted production rate to obtain the equivalent definition of the adjoint-weighted system lifetime written in terms of the adjoint-weighted neutron production rate and the effective multiplication factor, κ^+.

$$\tau_s^+ = \frac{\kappa^+ \int \frac{\Phi^+\Psi}{v}\, d\Omega dV dE}{\int \Phi^+ \chi_f \bar{\nu}_t \Sigma'_f \Psi'\, d\Omega dE' dV dE + \int \Phi^+ \chi_{2n} 2\Sigma'_{2n} \Psi'\, d\Omega dE' dV dE + \ldots} \quad , \tag{84}$$

Note that when the (*n*,*2n*), (*n*,*3n*), ..., etc. production sources are negligible, Eq. (84) reduces to the very familiar transport expression for the adjoint-weighted system lifetime,

$$\tau_s^+ = \frac{k_{eff} \int \frac{\Phi^+\Psi}{v}\, d\Omega dV dE}{\int \Phi^+ \chi_f \bar{\nu}_t \Sigma'_f \Psi'\, d\Omega dE' dV dE} \quad . \tag{85}$$

Using the same arguments presented in Section V, we can now define an adjoint-weighted absorption lifetime as

$$\frac{N^+}{\tau_a^+} = \int \Phi^+ \Sigma_a \Psi\, d\Omega dV dE \quad . \tag{86}$$

By taking the ratio of Eq. (83) to Eq. (86), we obtain

$$\tau_a^+ = \frac{\tau_s^+}{p_a} \quad , \tag{87}$$

where P_a is the adjoint-weighted absorption probability.

$$p_a = \frac{\int \Phi^+ \Sigma_a \Psi \; d\Omega dV dE}{\int \Phi^+ \nabla \bullet \boldsymbol{J} \; d\Omega dV dE + \int \Phi^+ \Sigma_a \Psi \; d\Omega dV dE} \quad . \tag{88}$$

Following the methodology, we can define an adjoint-weighted leakage lifetime as

$$\tau_l^+ = \frac{\tau_s^+}{p_l} \quad , \tag{89}$$

where p_l is the adjoint-weighted leakage probability. And, as before, the reciprocal of the adjoint-weighted system lifetime is the harmonic sum of the constituent lifetimes. That is,

$$\frac{1}{\tau_s^+} = \frac{1}{\tau_a^+} + \frac{1}{\tau_l^+} \quad . \tag{90}$$

IX. GENERATION TIME

Using the definitions presented in the previous sections, we can rewrite the time-dependent transport equation (integrated over angle, energy, and space) in the simple form,

$$\frac{dN^+}{dt} = \kappa^+ \left(\frac{N^+}{\tau_s^+} \right) + S^+ - \left(\frac{N^+}{\tau_s^+} \right) \quad . \tag{91}$$

The first term on the right-hand side of Eq. (91) represents the rate at which neutrons are produced by fission, (*n,2n*), (*n,3n*), ..., reactions, etc.; the second term represents the rate at which neutrons appear in the system due to intrinsic/external neutron sources; and the third term represents the rate at which neutrons are being lost from the system due to leakage and absorption.

Even though Eq. (91) balances adjoint-weighted neutrons, it does not explicitly account for the time-dependent rate at which delayed neutrons appear in the system following a fission. To properly account for delayed neutrons, we need to rewrite Eq. (91) as a system of coupled, differ-

ential equations.

$$\frac{dN^+}{dt} = \kappa^+(1-\beta')\left(\frac{N^+}{\tau_s^+}\right) + \sum \lambda_i C_i^+ + S^+ - \left(\frac{N^+}{\tau_s^+}\right) \quad , \tag{92}$$

and

$$\frac{dC_i^+}{dt} = \kappa^+\beta'_i\left(\frac{N^+}{\tau_s^+}\right) - \lambda_i C_i^+ \qquad \text{for } i=1,m \tag{93}$$

where C_i^+ is the adjoint-weighted precursor concentration, λ_i is the decay constant of the i^{th} precursor group, m is the number of delayed neutron groups, and β'_i is the reduced, adjoint-weighted delayed neutron fraction of the i^{th} precursor group. The first term on the right-hand side of Eq. (92) represents the number of *prompt* neutrons produced per unit time; the second term represents the rate at which delayed neutrons from all precursor groups are appearing in the system; the third term is the adjoint-weighted source rate; and, the fourth term is the total adjoint-weighted loss rate. Similarly, the first term on the right-hand side of Eq. (93) represents the rate at which delayed neutrons in the i^{th} delayed neutron group are produced by fission, and the second term represents the rate at which delayed neutrons are born due to precursor decay.

We can rewrite Eq. (92) as,

$$\frac{dN^+}{dt} = \left(\frac{\kappa^+(1-\beta')-1}{\tau_s^+}\right)N^+ + \sum \lambda_i C_i^+ + S^+ \quad . \tag{94}$$

If we define reactivity, ρ, as

$$\rho = \frac{\kappa^+ - 1}{\kappa^+} \quad , \tag{95}$$

and an adjoint-weighted neutron generation time, Λ_s^+, as

$$\Lambda_s^+ = \frac{\tau_s^+}{\kappa^+} \quad , \tag{96}$$

then Eqs. (92) and (93) can be rewritten as

$$\frac{dN^+}{dt} = \left(\frac{\rho-\beta'}{\Lambda_s^+}\right)N^+ + \sum \lambda_i C_i^+ + S^+ \quad , \tag{97}$$

and

$$\frac{dC_i^+}{dt} = \beta'_i \frac{N^+}{\Lambda_s^+} - \lambda_i C_i^+ \quad \text{for } i=1,m. \tag{98}$$

This system of equations is the one-region, point kinetic model based on the instantaneous multiplication factor κ rather than k_{eff}. Note that when the $(n,2n)$, $(n,3n)$, ..., etc. reactions within a particular system are negligible, then $\kappa = k_{eff}$, $\beta' = \beta_{eff}$, and $\rho = (k_{eff} - 1)/k_{eff}$; hence, Eqs. (97) and (98) collapse back to the traditional one-region, point kinetic model.

The one-region, point kinetic model is usually written in terms of reactivity, ρ, and the adjoint-weighted neutron generation time rather than the effective multiplication factor and the adjoint-weighted system lifetime. This stems from the fact that in the vicinity of delayed critical, Λ_s^+ is nearly constant, whereas, τ_s^+ varies as $\tau_{so}^+ k_{eff}$, where τ_{so} is the system lifetime at delayed critical. Hence, by writing the point kinetic model in terms of ρ and Λ_s^+, the dynamic behavior of the system can sometimes be described using the same set of coupled differential equations with the quite reasonable assumption that the coefficients are constant in the vicinity of delayed critical.

Using Eq. (84) and the definition of the adjoint-weighted system lifetime, we note that Λ_r corresponds to

$$\Lambda_s^+ = \frac{\int \frac{\Phi^+ \Psi}{v} \, d\Omega dV dE}{\kappa[\int \Phi^+ \nabla \bullet \boldsymbol{J} \, d\Omega dV dE + \int \Phi^+ \Sigma_a \Psi \, d\Omega dV dE]} \quad , \tag{99}$$

or, as,

$$\Lambda_s^+ = \frac{\int \frac{\Phi^+ \Psi}{v} \, d\Omega dV dE}{\int \Phi^+ \chi_f \bar{\nu}_t \Sigma'_f \Psi' \, d\Omega dE' dV dE + \int \Phi^+ \chi_{2n} 2\Sigma'_{2n} \Psi' \, d\Omega dE' dV dE + \ldots} \quad . \tag{100}$$

As with the adjoint-weighted system lifetime, if the $(n,2n)$, $(n,3n)$, ..., reactions are negligible, then Eq. (100) reduces to the very familiar transport equation for the neutron generation time,

$$\Lambda_s^+ = \frac{\int \frac{\Phi^+ \Psi}{v} \, d\Omega dV dE}{\int \Phi^+ \chi_f \bar{\nu}_t \Sigma'_f \Psi' \, d\Omega dE' dV dE} \quad . \tag{101}$$

The physical interpretation of Λ_s^+ is best seen by rewriting Eq. (100) as,

$$\left(\frac{N^+}{\Lambda_s^+}\right) = \int \Phi^+ \chi_f \bar{\nu}_t \Sigma'_f \Psi' \, d\Omega dE' dV dE + \int \Phi^+ \chi_{2n} 2\Sigma'_{2n} \Psi' \, d\Omega dE' dV dE + \ldots \quad . \tag{102}$$

From this equation we note that Λ_s^+ is the mean time between the appearance of a *production* neutron (per unit neutron population), whereas, the adjoint-weighted system lifetime is the mean time between the *loss* of a neutron due to leakage or absorption. When the effective multiplication factor is less than one, neutrons are being lost at a rate that is faster than the rate at which they are being produced; consequently, the neutron generation time will be larger than the neutron system lifetime. When the multiplication factor is greater than one, the opposite is true; and, at delayed critical, the generation time and the system lifetime must be equal since neutrons are being produced and lost at the same rate.

X. LIFETIME ALGORITHMS IN MONTE CARLO CODES

In the previous sections, we presented a set of equations that define a wide variety of unweighted and adjoint-weighted system lifetimes and/or neutron generation times written in terms of the angular–, energy–, and spatially–dependent neutron and adjoint fluxes. To evaluate these equations, one must obviously know these fluxes. For simple geometries, these fluxes can be estimated using a deterministic code which provides a rather complete description of the fluxes throughout the system. A Monte Carlo code does not include such fine detail but instead provides integral values for various reaction rates within the system from which an estimate of the unweighted system lifetime can be determined. Using more advanced time-dependent Monte Carlo techniques, one can even determine the adjoint-weighted system lifetime. Furthermore, Monte Carlo codes are not confined to simple geometries. Hence, it is worthwhile to understand the lifetime algorithms used in Monte Carlo codes so that the meaning of the lifetime values can be properly interpreted. In this section, we describe two different lifetime algorithms that are commonly used in Monte Carlo codes that yield estimates of the unweighted system lifetime—the last-event estimator and the collision estimator.

Last-Event Estimator

The last-event estimator tracks each neutron until it is lost from the system due to either absorption or leakage. Other collision events, such as elastic and inelastic scatterings, that occur prior to absorption or leakage do not add to the absorption or leakage score—only the last event is counted (i.e., the absorption or the leakage). Two tallies are maintained in this method; the first tally contains the life span of those neutrons that were absorbed and the second tally contains the life span of those neutrons that leaked.The average absorption life span, $\bar{t}_a$, is calculated as

$$\bar{t}_a = \frac{1}{N_a} \sum_{n=1}^{N_a} T_n \quad , \tag{103}$$

where T_n is the total length of time from birth to absorption and N_a is the total number of particles that were absorbed.The average leakage life span, $\bar{t}_l$, is calculated as

$$\bar{t}_l = \frac{1}{N_l} \sum_{n=1}^{N_l} T_n \quad , \tag{104}$$

where N_l is the total number of particles that leaked from the system.

Based on these two average life spans and the fraction of neutrons involved in each type of event, the average, unweighted system lifetime is estimated by

$$\tau_s = p_a \bar{t}_a + p_l \bar{t}_l \quad , \tag{105}$$

where p_a is the total fraction of neutrons that are lost in an absorption reaction, and p_l is the fraction of neutrons that leak from the system. The fractions p_a and p_l are related to N_a and N_l as

$$p_a = \frac{N_a}{N_a + N_l} \quad , \tag{106}$$

and

$$p_l = \frac{N_l}{N_a + N_l} \quad . \tag{107}$$

These definitions are equivalent to the expressions previously given in Eqs. (46) and (48).

From a physical standpoint, the average neutron life span for a particular process repre-

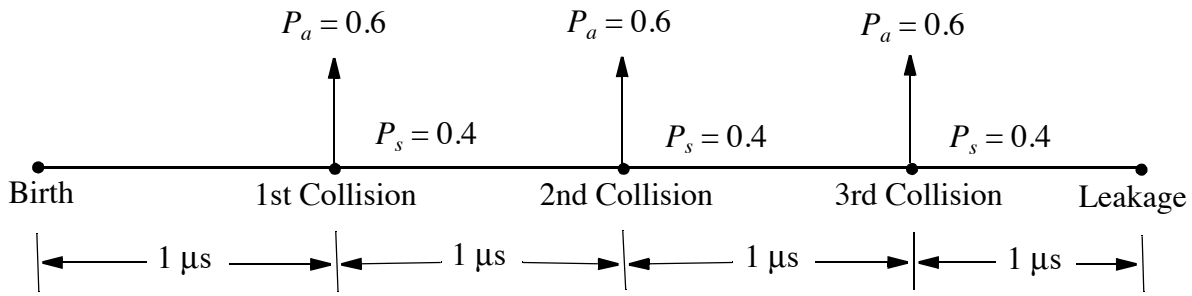

Fig. 1. Synoptic Diagram for Collision-Estimator Example.

sents the average life expectancy (i.e., time from birth to death) for a neutron to have a reaction of a particular type, whereas, the system lifetime is a measure of the mean time *per unit neutron population* between loss events.

Collision Estimator

In the collision-estimator method, neutron histories are not terminated following absorption. Each neutron is tracked until it either leaks out of the system, or drops below a predetermined weight cutoff value. The neutron lifetime is determined by summing the time required to reach the i^{th} collision point, weighted by the probability of reaching that collision point. In this fashion, each collision point contributes some information to the average lifetime estimate. To demonstrate this estimator, consider the following example.

Assume that a neutron is born at some random location somewhere in the system and, during a random walk, has three collisions prior to leaking (see Figure 1). Let's further assume that the time from birth to the first collision is 1 μs, and that the time between the first, second, and third collisions is also 1 μs. After the 3rd collision, we assume it takes another 1 μs for the neutron to reach a leakage surface. At each collision point, the probability of absorption, $P_a=\Sigma_a/\Sigma_t$, is 60% and the probability of scattering, $P_s=\Sigma_s/\Sigma_t$, is 40%. The average lifetime corresponding to this particular neutron history would be

$$\tau = (0.6 \times 1) + (0.4 \times 0.6 \times 2) + (0.4^2 \times 0.6 \times 3) + (0.4^3 \times 4) = 1.624 \quad .$$

The first term on the right-hand side of the above expression corresponds to the time elapsed since birth (i.e., 1 μs) multiplied by the probability of being absorbed on the first collision. The second term corresponds to the time elapsed since birth (i.e., 2 μs) multiplied by the probability of being scattered on the first collision and then being absorbed on the second collision. The three term corresponds to the time elapsed since birth (i.e., 3 μs) multiplied by the probability of being scattered on the first and second collision and then being absorbed on the third collision. And finally, the last term corresponds to the time elapsed since birth (i.e., 4 ms) multiplied by the probability of being scattered on the first, second, and third collisions and then leaking from the system.

We can generalize this lifetime calculation based on N total neutron histories.[f] For the n^{th} neutron history, the probability of an absorption at the i^{th} collision point, P_{ai}^n, corresponds to

$$P_{ai}^n = \frac{\Sigma_{ai}^n}{\Sigma_{ti}^n} \quad , \tag{108}$$

where Σ_{ai}^n is the macroscopic absorption cross section corresponding to the energy of the incident neutron, and Σ_{ti}^n is the total macroscopic cross section corresponding to the energy of the incident neutron. The number of neutrons not absorbed is, of course, the scattering probability, P_{si}^n, which is defined as

$$P_{si}^n = \frac{\Sigma_{si}^n}{\Sigma_{ti}^n} = 1 - P_{ai}^n \quad , \tag{109}$$

where Σ_{si}^n is the macroscopic scattering cross section corresponding to the energy of the incident neutron. The probabilities P_{ai}^n and P_{si}^n do not correspond to the previously defined probabilities p_a and p_l. The latter quantities, p_a and p_l, are relative to the total loss rate, whereas, P_{ai}^n and P_{si}^n are relative to the collision interaction rate.

The average lifetime corresponding to the n^{th} neutron history is obtained from the following finite series.

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f. The following derivation was a collaboration between G. D. Spriggs (LANL) and Lester M. Petrie (ORNL).

$$\tau^n = P^n_{a1}\Delta T^n_1 + P^n_{s1}P^n_{a2}(\Delta T^n_1 + \Delta T^n_2) + P^n_{s1}P^n_{s2}P^n_{a3}(\Delta T^n_1 + \Delta T^n_2 + \Delta T^n_3) + \ldots$$

$$P^n_{s1}P^n_{s2}\ldots P^n_{s(J_n-1)}P^n_{aJ_n}(\Delta T^n_1 + \Delta T^n_2 + \ldots + \Delta T^n_{J_n}) +$$

$$P^n_{s1}P^n_{s2}\ldots P^n_{sJ_n}(\Delta T^n_1 + \Delta T^n_2 + \ldots + \Delta T^n_{J_n} + \Delta T^n_{J_n+1}) \quad , \tag{110}$$

where ΔT^n_1 is the time from birth to the first collision, ΔT^n_2 is the time from the first collision to the second collision, and so forth, and J_n is the total number of collisions the particle had before leaking from the system.

For convenience, we define

$$T^n_1 = \Delta T^n_1 \quad ,$$

$$T^n_2 = \Delta T^n_1 + \Delta T^n_2 \quad ,$$

$$T^n_i = \Delta T^n_1 + \Delta T^n_2 + \ldots + \Delta T^n_i \quad ,$$

$$T^n_{J_n} = \sum_{i=1}^{J_n} \Delta T^n_i \quad , \tag{111}$$

and

$$W^n_1 = P^n_{s0} \equiv 1.0 \quad ,$$

$$W^n_2 = P^n_{s0} \cdot P^n_{s1} \quad ,$$

$$W^n_3 = P^n_{s0} \cdot P^n_{s1} \cdot P^n_{s2} \quad ,$$

$$W^n_i = \prod_{k=0}^{i-1} P^n_{sk} \quad . \tag{112}$$

Using these definitions, Equation (110) reduces to

$$\tau^n = W^n_1 P^n_{a1} T^n_1 + W^n_2 P^n_{a2} T^n_2 + \ldots + W^n_{J_n} P^n_{aJ_n} T^n_{J_n} + W^n_{J_n+1} T^n_{J_n+1} \quad , \tag{113}$$

which, in summation form, can be written as

$$\tau^n = \left| \sum_{i=1}^{J_n} W_i^n T_i^n P_{ai}^n \right| + W_{J_n+1}^n T_{J_n+1}^n \quad . \tag{114}$$

For N neutron histories, the average system lifetime is calculated by

$$\tau_s = \frac{1}{N} \sum_{n=1}^{N} \tau^n \quad , \tag{115}$$

or

$$\tau_s = \frac{1}{N} \sum_{n=1}^{N} \left\{ \left| \sum_{i=1}^{J_n} W_i^n T_i^n P_{ai}^n \right| + W_{J_n+1}^n T_{J_n+1}^n \right\} \quad . \tag{116}$$

We cannot present a formal proof that demonstrates that Eqs. (105) and (116) are equivalent to the unweighted system lifetime defined by Eq. (31). However, this is easily demonstrated by comparing results from a deterministic code and a Monte Carlo code. These results will be presented in a subsequent manuscript. Notwithstanding this formal proof, we can demonstrate that both Eqs. (105) and (116) reduce to the definition of the infinite-medium lifetime.

If we assume an infinite system, then the number of neutrons that leak will be zero. Hence, $p_l = 0$ and $p_a = 1.0$ in Eq. (105). Therefore, τ_s is identically equal to $\overline{t_a}$. A neutron traveling at the average velocity $\bar{v}$ will travel an average distance of $\overline{d}_a$ in the average time $\overline{t_a}$. By definition, $\overline{d}_a$ is equal to $1/\overline{\Sigma}_a$. So, τ_s is equal to

$$\tau_\infty = \frac{1}{\bar{v}\overline{\Sigma}_a} \quad . \tag{117}$$

Similarly, Equation (116) reduces to Eq. (117) if we assume no leakage (i.e., $J_n \to \infty$ for each n^{th} history) and further assume

$$P_{a1}^n = P_{a2}^n = P_{a3}^n = \ldots = \overline{P}_a \quad ,$$

$$P_{s1}^n = P_{s2}^n = P_{s3}^n = \ldots = \overline{P}_s \quad ,$$

where $\overline{P}_a + \overline{P}_s = 1$.

By assuming that the times between collisions are all identically equal to the average,

$$\Delta T_1^n = \Delta T_2^n = \Delta T_3^n = \ldots = \overline{\Delta T} \quad ,$$

then we can write $T_i = i\overline{\Delta T}$. Hence, Eq. (116) reduces to

$$\tau_\infty = \frac{1}{N}\sum_{n=1}^{N}\left|\left\{\overline{\Delta T}\sum_{i=1}^{\infty} i\overline{P}_a(1-\overline{P}_a)^{i-1}\right\} + \overline{\Delta T}\lim_{i\to\infty} i\overline{P}^{i}_{s}\right| \quad . \tag{118}$$

In the limit, the last term inside of the summation goes to zero and the infinite series converges to

$$\sum_{i=1}^{\infty} i\overline{P}_a(1-\overline{P}_a)^{i-1} = \frac{1}{\overline{P}_a} \quad . \tag{119}$$

Consequently, Eq. (118) becomes

$$\tau_\infty = \frac{1}{N}\sum_{n=1}^{N}\frac{\overline{\Delta T}}{\overline{P}_a} = \frac{\overline{\Delta T}}{\overline{P}_a} \quad , \tag{120}$$

where, by definition,

$$\overline{P}_a = \frac{\overline{\Sigma}_a}{\overline{\Sigma}_t} \quad . \tag{121}$$

We also note that the average time between collision is equal to the average distance traveled between collisions, $\overline{d}_t$, divided by the average velocity of the system neutrons.

$$\overline{\Delta T} = \frac{\overline{d}_t}{\overline{v}} \quad . \tag{122}$$

Since the average distance traveled between collisions is identically equal to $1/\overline{\Sigma}_t$, then it follows that

$$\tau_\infty = \frac{\bar{d}_t}{\bar{v}} \frac{\overline{\Sigma}_t}{\overline{\Sigma}_a} = \frac{1}{\bar{v}\overline{\Sigma}_a} \quad . \tag{123}$$

We mentioned earlier that the adjoint-weighted system lifetime can be ascertained using time-dependent Monte Carlo techniques. This will be discussed in more detail in a subsequent manuscript.

XI. CONCLUSIONS

In this work, a set of equations derived from the transport equation have been presented that define unweighted and adjoint-weighted lifetimes characterizing various reaction rates. We showed that a lifetime τ_i, defined in the general form,

$$\frac{N}{\tau_i} = I_i \quad ,$$

represents the mean time per unit neutron population between interactions of the i^{th} kind.

We have also shown that the α-eigenvalue corresponding to the asymptotic solution of the transport equation can be written in the general form,

$$\alpha\tau_x = x - 1 \quad ,$$

where x corresponds to the instantaneous value of the x-eigenvalue. When x corresponds to κ, τ_x corresponds to the system lifetime; when x corresponds to γ, τ_x corresponds to the collision lifetime; and so forth. Since it is more customary to work with k-eigenvalues in reactor physics applications and criticality safety, we have concentrated on defining the system lifetime—which represents the mean time per unit neutron population between loss events resulting from absorption and leakage. In addition, we demonstrated how other lifetimes can be formulated in terms of the system lifetime.

And finally, we have described two different lifetime estimators used in Monte Carlo codes. The last-event estimator is based on the determination of the absorption and leakage life

spans. A neutron life span represents the mean time from birth-to-event, whereas, a neutron life-time represents the mean time per unit neutron population from event-to-event.

REFERENCES

1. O. R. FRISCH and R. PEIERLS, "Memorandum on the Properties of a Radioactive Super-bomb," (1940); reproduced in book by Ronald W. Clark, Tizard, MIT Press (1965); also reproduced in book by Robert Serber, The Los Alamos Primer, Univ. of Calif. Press, Berkely, CA (1992).

2. SERBER, ROBERT, "The Los Alamos Primer," Los Alamos National Laboratory unclassified report LA-1 (April 1943).

3. FEYNMAN, R. P., "Measurement of Initial Multiplication Rate of Supercritical Gadgets," Los Alamos National Laboratory classified report LA-29 (October 14, 1943).

4. de HOFFMANN, F. et al., "Criticality of the Water Boiler, Number of Delayed Neutrons, and Dispersion of the Neutron Emission per Fission," Los Alamos National Laboratory unclassified report LA-183 (December 8, 1944).

5. KUPFERBERG, KENNETH M. et al., "Lifetime of Neutrons in Tampers," Los Alamos National Laboratory classified report, LA-530 (March 22, 1946).

6. FEYNMAN, R. P., "Statistical Behavior of Neutron Chains," Los Alamos National Laboratory unclassified report LA-591 (July 26, 1946).

7. BAKER, C. P., "Time Scale Measurements by the Rossi Method," Los Alamos National Laboratory unclassified report LA-617 (January 24, 1947).

8. HURWITZ, HENRY, JR., "Derivation and Integration of the Pile-Kinetic Equations," *Nucleonics*, **5**, 61 (July 1949).

9. GLASSTONE, SAMUEL and MILTON C. EDLUND, The Elements of Nuclear Reactor Theory, Princeton, NJ, 1952.

10. WEINBERG, ALVIN M., "Current Status of Nuclear Reactor Theory," *Am. J. Physics*, **20** (7), 401 (1952).

11. USSACHOFF, L. N., "Equation for the Importance of Neutrons, Reactor Kinetics and the Theory of Perturbation," *Proc. 1st Intern. Conf. Peaceful Uses Atomic Energy*, Geneva, Switzerland, **5**, 503 (1955).

12. WEINBERG, ALVIN M. and EUGENE P. WIGNER, The Physical Theory of Neutron Chain Reactors, Chicago, IL, 1958.

13. LEWINS, JEFFERY, "The Use of the Generation Time in Reactor Kinetics," *Nucl Sci. Engr.*, **7**, 122 (1960).

14. NISHIHARA, HIROSHI and SEIJI UKAI, "A Study on the Space and Energy Dependent Reactor Kinetics, with Direct Physical Interpretation of the Effective Neutron Lifetime and Criticality Factor," *Memoirs of the Faculty of Eng. Kyoto Univ.*, **25** (2), 264 (1963).

15. ASAOKA, T. et al., "Prompt-Neutron Lifetime Calculations in Fast Assemblies," *Trans. Am. Nucl. Soc.*, **7**, 514 (1964).

16. RIEF, H. H. KSCHWENDT, "Reactor Analysis by Monte Carlo," *Nucl Sci. Engr.*, **30**, 395 (1967).

17. NELSON, PAUL, JR., "Relationships Between Various Definitions of Prompt-Neutron Lifetime," *Nucl Sci. Engr.*, **43**, 154 (1971).

18. MAROTTA, CHARLES R., "Neutron Utilization Parameter for Complex Coupled Systems," *Nucl. Sci. & Eng.*, **77**, 107 (1981).

19. MAROTTA, CHARLES R., "Response to 'Neutron Lifetime, Generation Time, and Reproduction Time'," letter to editor *Nucl. Sci. & Eng.*, **78**, 106 (1981).

20. LEWINS, JEFFERY D., "Neutron Lifetime, Generation Time, and Reproduction Time," letter to editor *Nucl. Sci. & Eng.*, **78**, 105 (1981).

21. HAYASHI, MASATOSHI, "Comments of 'Neutron Lifetime, Fission Time and Generation Time'," Letter to the Editor, *Nucl. Sci. & Eng.*, **121**, No. 3, 492 (Nov. 1995).

22. BELL, GEORGE I. and SAMUEL GLASSTONE, Nuclear Reactor Theory, Malabar, FL, 1985.

23. BRIESMEISTER, JUDITH F., ed., "MCNP—A General Monte Carlo N-Particle Transport Code, Version 4A," Los Alamos National Laboratory report LA-12625-M (Nov. 1993).

24. PETRIE, L. M. and N. F. CROSS, "KENO IV, An Improved Monte Carlo Criticality Program," ORNL-4938, Oak Ridge National Laboratory (Nov. 1975).

25. RONEN, Y., D. SHVARTS, and J. J. WAGSCHAL, "A Comparison of Some Eigenvalues in Reactor Theory," *Nucl. Sci. & Eng.*, **60**, 97 (1976).

26. SHALITIN, D., J. J. WAGSCHAL, AND Y. YEIVIN, "A Theorem on the Validity of the

One-Group Transport Equation," *Trans. Am. Nucl. Soc.*, 23, 539 (1976).

27. RONEN, Y., D. SHALITIN, AND J. J. WAGSCHAL, "A Useful Different Eigenvalue for the Transport Equation," *Trans. Am. Nucl. Soc.*, 24, 474 (1976).

28. SHALITIN, D., J. J. WAGSCHAL, AND Y. YEIVIN, "Neutron Energy Spectra in Different Formulations of the Stationary Transport Equation," *Trans. Am. Nucl. Soc.*, 27, 945 (1977).

29. VELARDE, GUILLERMO, CAROLINA AHNERT, AND JOSE M. ARAGONES, "Analysis of the Eigenvalue Equations in k, λ, γ, and α Applied to Some Fast– and Thermal–Neutron Systems," *Nucl. Sci. & Eng.*, **66**, 284 (1978).

30. CACUCI, D. G. et. al, "Eigenvalue-Dependent Neutron Energy Spectra: Definitions, Analyses, and Applications," *Nucl. Sci. & Eng.*, **81**, 432 (1982).

31. LITTLE, ROBERT, private communication (1996).

32. R. AVERY et al., "Coupled Fast-thermal Power Breeder Critical Experiment," *Proc. 2nd Intern. Conf. Peaceful Uses Atomic Energy*, Geneva, Switzerland, **12**, 151 (1958).

33. R. N. CORDY, "Measurement of the Kinetic Experiment Water Boiler Reactor Transfer Function by Reactor Modulation Techniques," *Trans. Amer. Nucl. Soc.*, **3**, 1, 120, (1960).

34. P. T. HANSSON AND L. R. FOULKE, "Investigations in Spatial Reactor Kinetics," *Nucl. Sci. & Eng.*, **17**, 528 (1963).

35. R. A. KARAM, "Measurement of Rossi-Alpha in Reflected Reactors," *Trans. Amer. Nucl. Soc.*, **7**, 2, 283, (1964).

36. S. R. BIERMAN and E. D. CLAYTON, "Pulsed Neutron Source and Reactor Noise Studies of Plutonium Systems," *IAEE Proc. of Symp. on Pulsed Neutron Research*, Karlsruhe, Germany, STI/PUB/104, **2**, 825 (1965).

37. R. A. KARAM, "Spatial Dependence of the Decay Rates of Prompt-Neutron Chains in Reflected Reactors," *Trans. Amer. Nucl. Soc.*, **8**, 1, 224, (1965).

38. G. S. BRUNSON and R. J. HUBER, "Pulsed-Neutron Die-Away Measurements in Three Fast Critical Assemblies," *Trans. Amer. Nucl. Soc.*, **9**, 1, 168, (1966).

39. G. M. WELLS, "Pulsed-Neutron Measurements in Far-Subcritical Multiplying Media With and Without Large Reflectors," *Trans. Amer. Nucl. Soc.*, **9**, 1, 170, (1966).

40. J. F. KUNZE and F. L. SIMS, "Neutron Lifetime Measurements on a Two-Lifetime Component Critical Experiment by the Rossi-Alpha Technique," General Electric Company

report GEMP-281 (April 6, 1964); also "Rossi-Alpha Measurements on a Multiple-Component Lifetime System," *Trans. Amer. Nucl. Soc.*, **9**, 2, 467, (1966).

41. M. H. McTAGGART et al., "The Fast Pulsed Reactor VIPER: Reactor Physics measurements and Analysis," *J. of British Nucl. Energy Soc.*, **7**, 4, 328 (1968).

42. F. N. McDONNELL and M. J. HARRIS, "Pulsed-Source Experiments in a Reflected Coupled-Core Reactor," *Trans. Amer. Nucl. Soc.*, **14**, 2, 864, (1971).

43. G. S. BRUNSON and R. J. HUBER, "Two-Region Analysis of Pulsing Data in Fast Critical Systems," *Nucl. Inst. and Methods*, **128**, 379 (1975).

44. D. L. HETRICK and D. P. GAMBLE, "Transient Reactivity During Power Excursions in a Water Boiler Reactor," *Trans. Amer. Nucl. Soc.*, **1**, 2, 48, (1958).

45. D. P. GAMBLE, "The Effect of Reflector-Moderated Neutrons on the Kinetics of the Kinetic Experiment Water Boiler Reactor," *Trans. Amer. Nucl. Soc.*, **3**, 1, 122, (1960).

46. M. S. DUNENFELD, "Analyses of the Dynamics of the Kinetic Experiment Water Boiler Reactor," *Trans. Amer. Nucl. Soc.*, **4**, 1, 70, (1961).

47. R. L. LONG, "Effects of Reflectors on the Burst Characteristics of the White Sands Missile Range (WSMR) Fast-Burst Reactor," *Trans. Amer. Nucl. Soc.*, **8**, 2, 451, (1965).

48. R. L. COATS, "Kinetic Behavior of a Reflected Fast-Burst Reactor," *Trans. Amer. Nucl. Soc.*, **9**, 2, 468, (1966).

49. R. L. COATS and P. D. O'BRIEN, "SPR-II Safety Analysis Report," SC-RR-66-2706, Sandia Corporation, Alb., NM (January 1967).

50. R. L. COATS, "Neutronic Decoupling of Fast-Burst Reactors," *Trans. Amer. Nucl. Soc.*, **10**, 1, 243, (1967).

51. C. C. PRICE, "Prompt Neutron Decay Constants in a Reflected Fast Burst Reactor," Dissertation, University of New Mexico (1970).

52. SPRIGGS, GREGORY D., ROBERT D. BUSCH and JOHN G. WILLIAMS, "Two-Region Kinetic Models for Reflected Reactors," *Annals of Nucl. Energy*, (1996).

53. McLANE, V., C. L. DUNFORD, and P. F. ROSE, "Data Formats and Procedures for the Evaluated Nuclear Data File ENDF-6," Brookhaven National Laboratory report BNL-NCS-44945, ENDF-102 (Rev. Nov. 1995).

54. ALCOUFFE, RAY E. et al., "DANTSYS: A Diffusion Accelerated Neutral Particle Transport Code System," Los Alamos National Laboratory report LA-12969-M (June 1995).

55. SPANIER, JEROME and ELY M. GELBARD, Monte Carlo Principles and Neutron Transport Problems, Reading, MA, 1969.

56. L. L. CARTER and E. D. CASHWELL, Particle-Transport Simulation with the Monte Carlo Method, Technical Information Center, Office of Public Affairs, U. S. Energy Research and Development Administration, TID-26607, 1975.

57. DILLENBURG, DARCY et al., "The $Be^9(n,2n)Be^8$ Reaction and its Influence on the Infinite Multiplication Factor for Beryllium Moderated Heterogeneous Thermal Reactors," *Proc. of 2nd United Nations International Conference on the Peaceful Uses of Atomic Energy*, **12**, 607 (1958).

58. KRASIN, A. K. et al., "Physical Characteristics of Beryllium Moderated Reactor," *Proc. of 2nd United Nations International Conference on the Peaceful Uses of Atomic Energy*, **12**, 571 (1958).

59. JAIN, R. D. and K. R. SRINVASAN, "Nuclear Study of a Low Power Beryllium Oxide Natural Uranium Power Reactor," *Proc. of 2nd United Nations International Conference on the Peaceful Uses of Atomic Energy*, **12**, 580 (1958).

60. BENOIST, P. et al., "Critical and Subcritical Experiments on U-BeO Lattices," *Proc. of 2nd United Nations International Conference on the Peaceful Uses of Atomic Energy*, **12**, 585 (1958).

61. MISAWA, TSUYOSHI, SEIJI SHIROYA, and KEIJI KANDA, "Study on Reactivity Worth of Beryllium by $(n,2n)$ and (γ,n) Reactions," *Nucl. Tech.*, **116**, 9 (Oct. 1996).